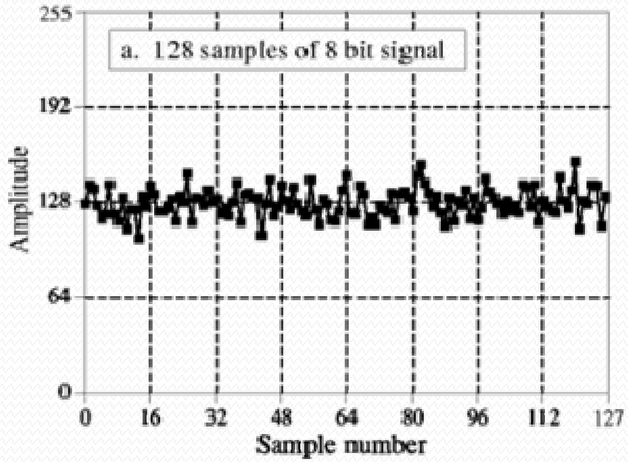
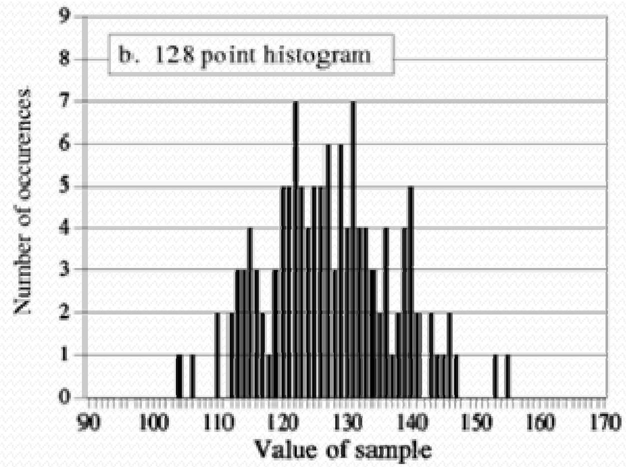
EE327 Digital Signal Processing ADC and DAC

Yasser F. O. Mohammad

REMINDER 1 Histogram (acquired signal)

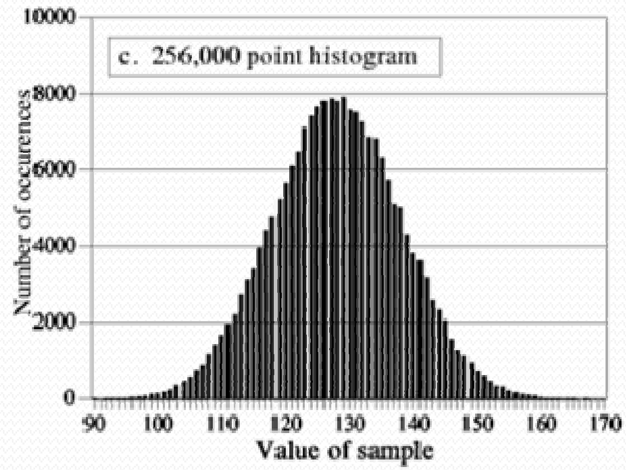
$$N = \sum_{i=0}^{M-1} H_i$$





$$\mu = \frac{1}{N} \sum_{i=0}^{M-1} i H_i$$

$$\sigma^2 = \frac{1}{N-1} \sum_{i=0}^{M-1} (i-\mu)^2 H_i$$



REMINDER 2:The Mighty Gaussian

$$P(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

REMINDER 3: How to combine random signals

- Assume X, Y are INDEPENDENT random signals with mean μ_x , μ_y and std. dev. σ_x, σ_y :

$$\mu_{aX \pm b} = a\mu_X \pm b \qquad \sigma^2_{aX \pm b} = a^2\sigma^2_X$$

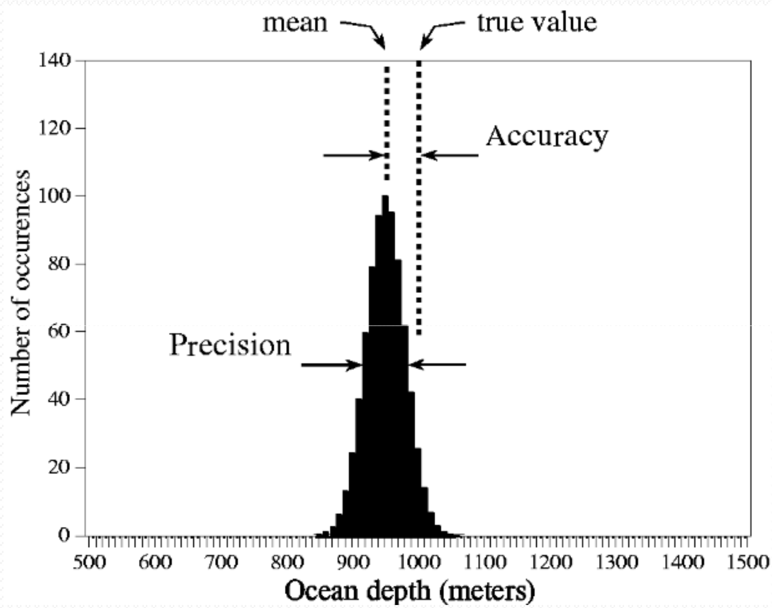
$$\mu_{aX \pm bY} = a\mu_X \pm b\mu_Y \qquad \sigma^2_{aX \pm bY} = a^2\sigma^2_X + b^2\sigma^2_Y$$

SELF Test: Prove these identities

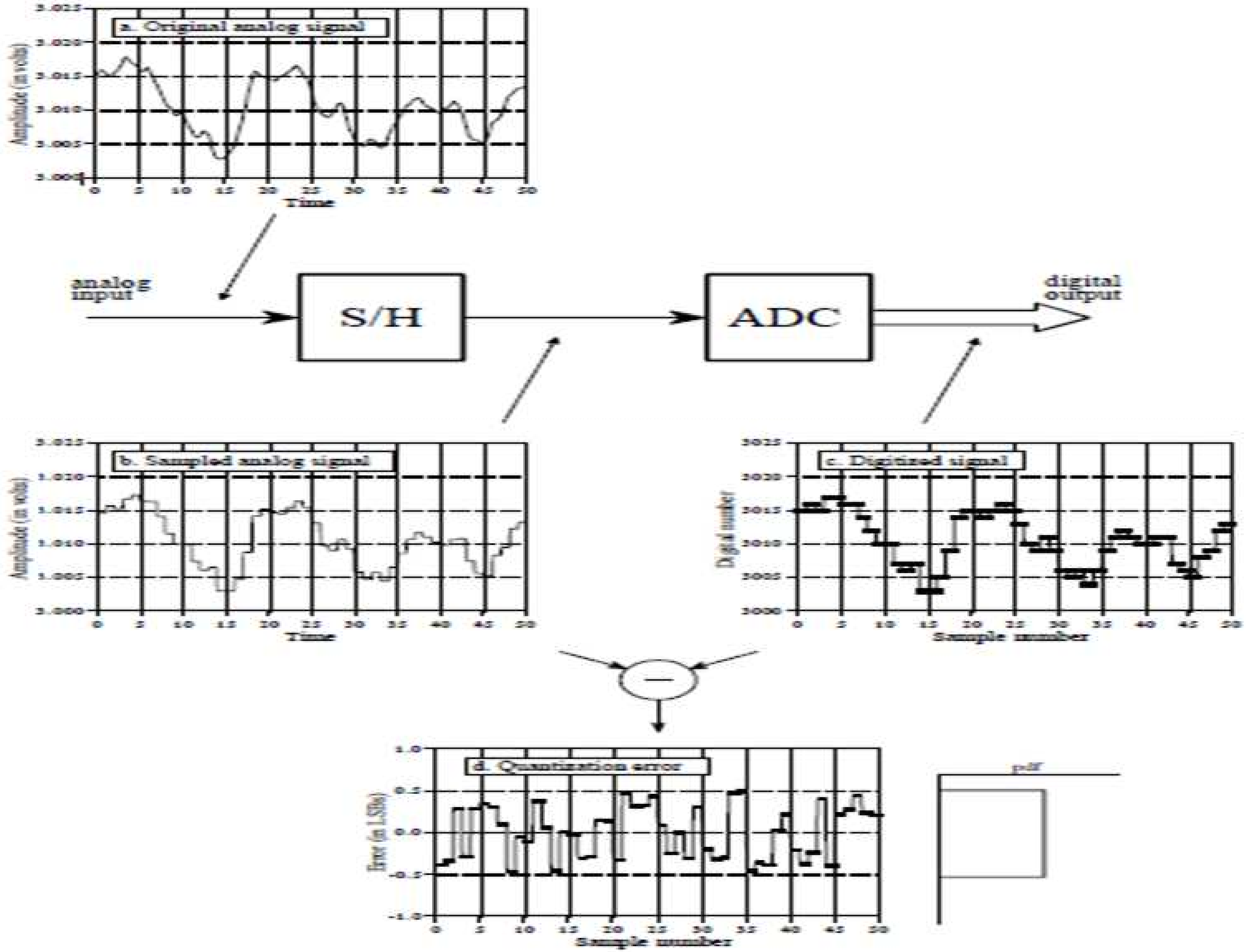
REMINDER 4: Precision and Accuracy

- Precision = repeatability
- Accuracy = bias

  → systematic errors
- The two questions:
  - Repeating will remove the error → precision
  - Calibration will remove the error → accuracy



ADC



a. Original analog signal
Amplitude (in volts)
Time
analog input
S/H
ADC
digital output
b. Sampled analog signal
Amplitude (in volts)
Time
c. Digitized signal
Digital number
Sample number
d. Quantization error
Error (in LSBs)
Sample number
pdf

Sampling and Quantization

- S/H
  - Sampling
  - Discretizes the independent variable
- ADC
  - Quantization
  - Discretizes the dependent variable

Error Due To Quantization

- ±½LSB
- Additive Error

- Continuous = Quantized + Quantization Error

- Uniform Distribution with mean of zero and standard deviation of ($LSB/\sqrt{12} \approx 0.29LSB$)

- Depends on #Bits and controls precision

Quantization Example

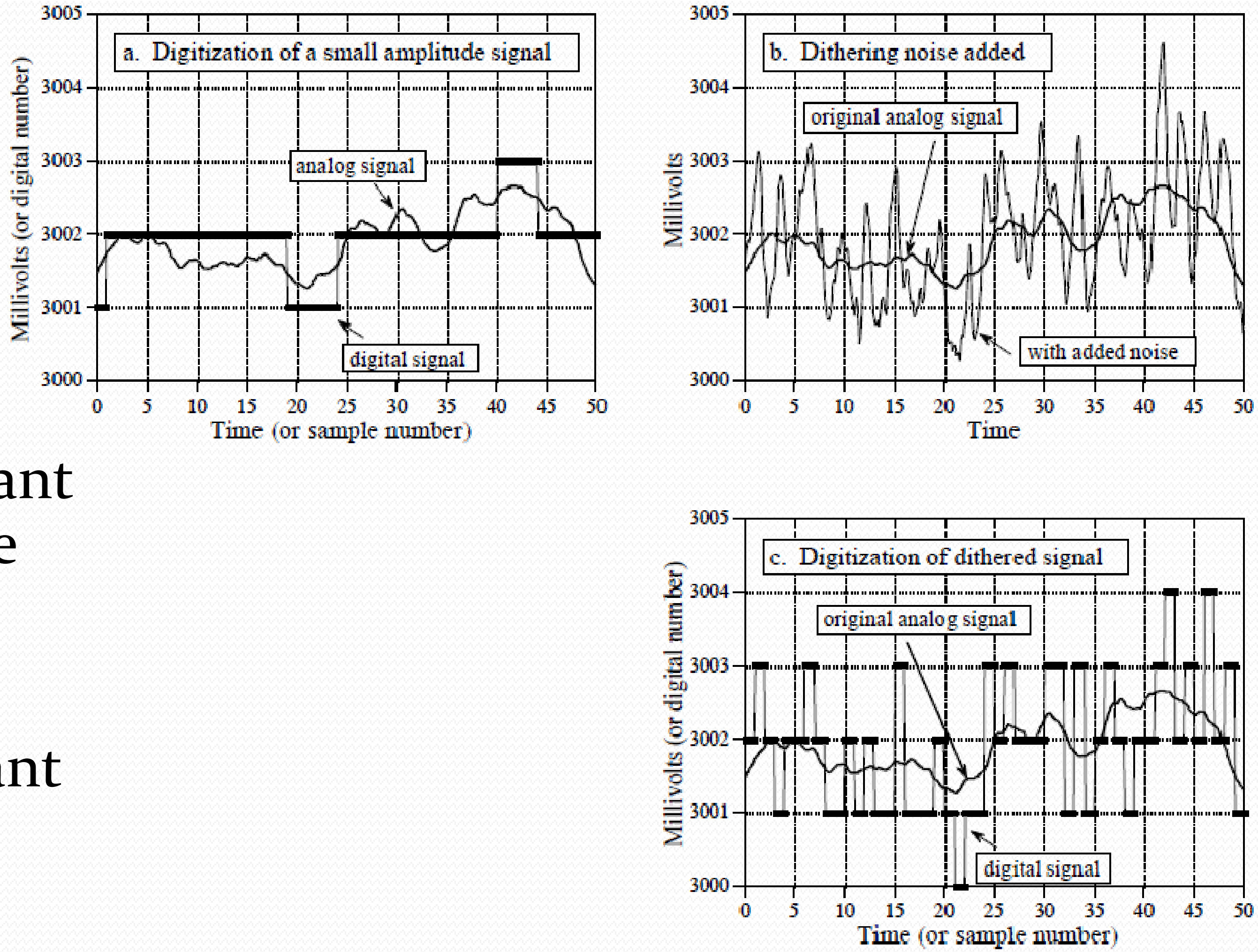
- Signal Amplitude = 1.0 Volt
- Uniformly random noise = 1 millivolt RMS
- 8 Bit Digitizer

- Noise = 0.255 LSB

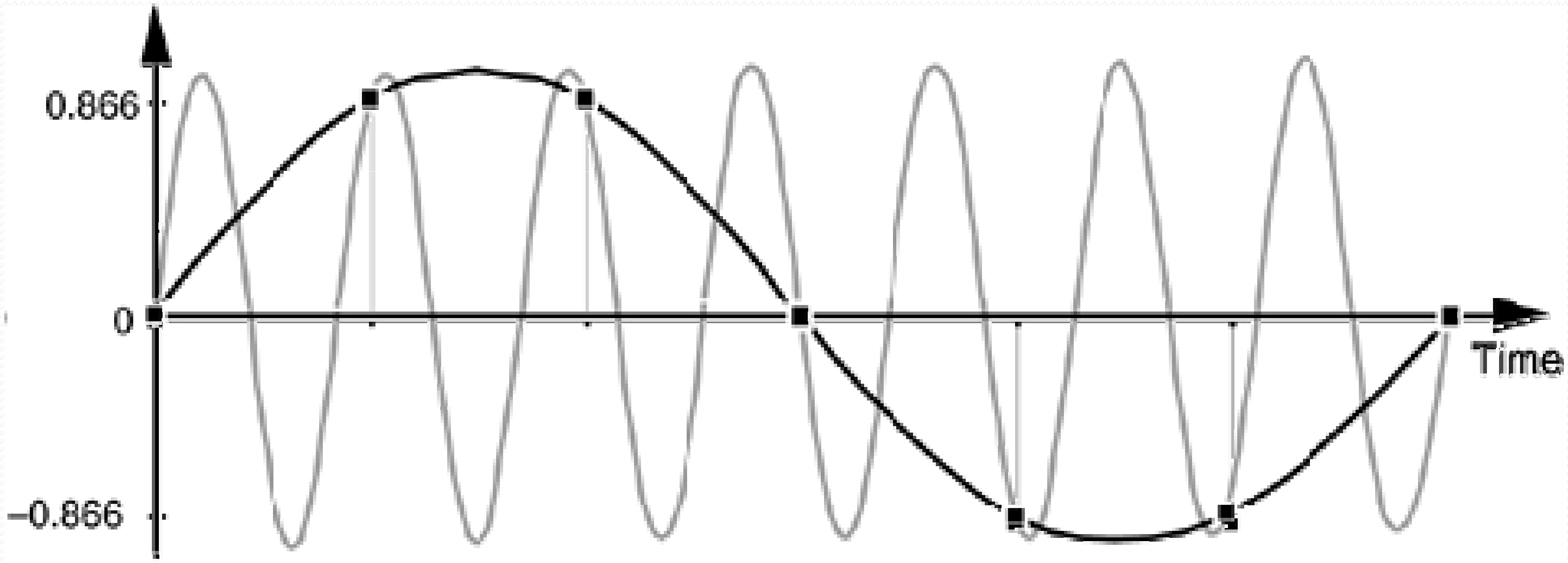
- Noise + Quantization = $\sqrt{0.255^2 + 0.29^2}\,LSB = 0.386 LSB$
- = 0.386/255= V
- 50% increase in noise

Dithering

- Add noise to reduce quantization error !!!!!
- Used when input is constant for a long time
- Quantization error is constant and additive



Sampling Example



Sampling Example (math)

$$x(t) = \sin(2\pi f_0 t)$$

$$x[n] = \sin(2\pi f_0 n t_s)$$

$$x[n] = \sin(2\pi f_0 n t_s + 2m\pi)$$

$$x[n] = \sin\left(2\pi\left(f_0 + \frac{m}{n t_s}\right) n t_s\right)$$

$$= \sin\left(2\pi\left(f_0 + k f_s\right) n t_s\right)$$

$$\therefore \sin(2\pi\left(f_0\right) n t_s) = \sin\left(2\pi\left(f_0 + k f_s\right) n t_s\right) \text{ for any integer } k$$

Is this limited to sinusoidals?

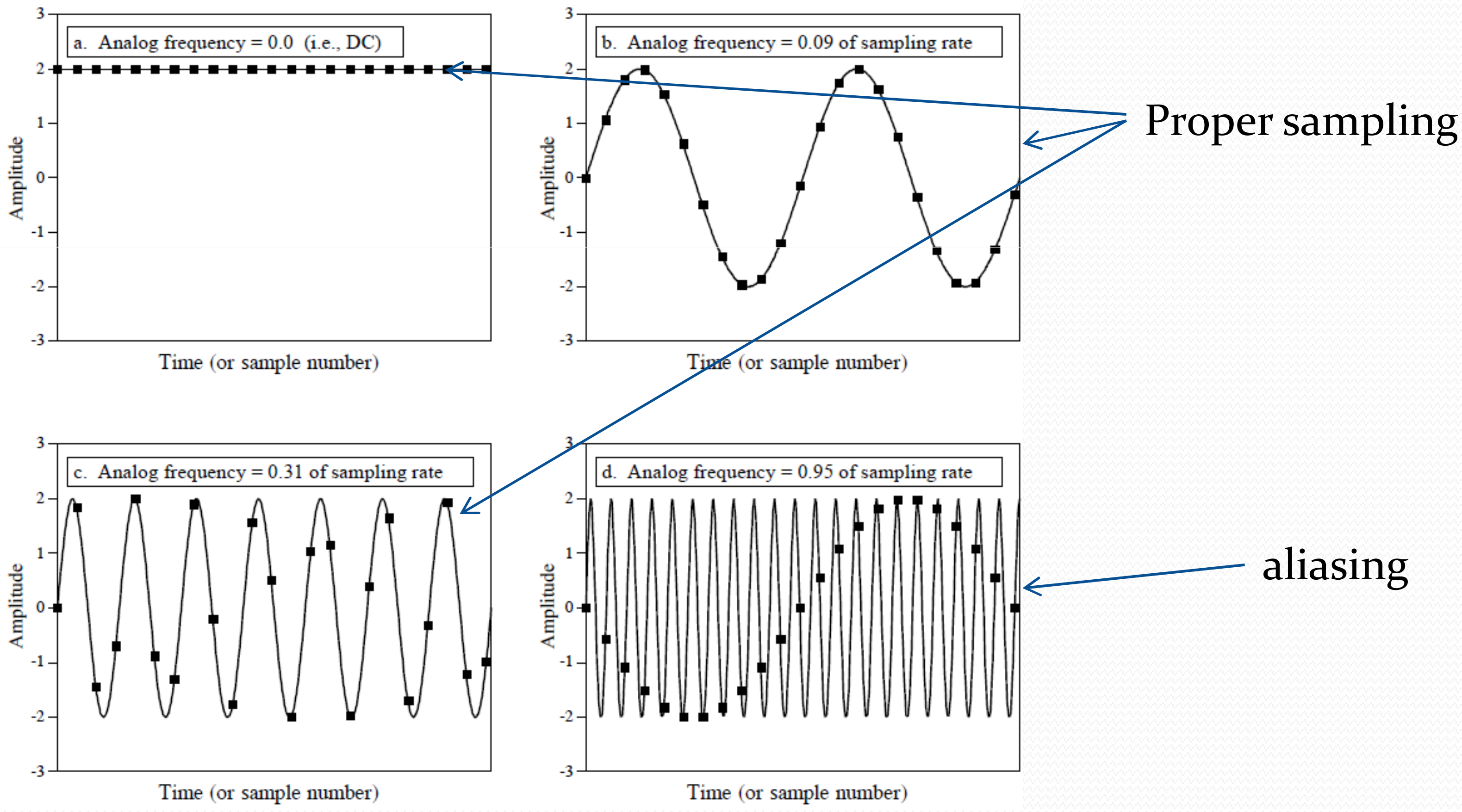
- Yes and No!!!!
- Every signal can be approximated to infinite accuracy using a set of sinusoidals:

- Periodic Time Domain → Discrete Frequency Domain
- Discrete Time Domain → Periodic Frequency Domain

		Periodicity	
Continuity		Periodic	aperiodic
	continuous	Fourier Series Aperiodic Spectrum Discrete Spectrum	Fourier Transform Aperiodic Spectrum Continuous Spectrum
	discrete	Discrete Fourier Transform Periodic Spectrum Discrete Spectrum	Discrete Fourier Transform Periodic Spectrum Continuous Spectrum

Sampling

- Our goal is to be able to reconstruct the analog signals completely from the digitized version (ignoring quantization).



Nyquist Frequency

- Half the sampling rate

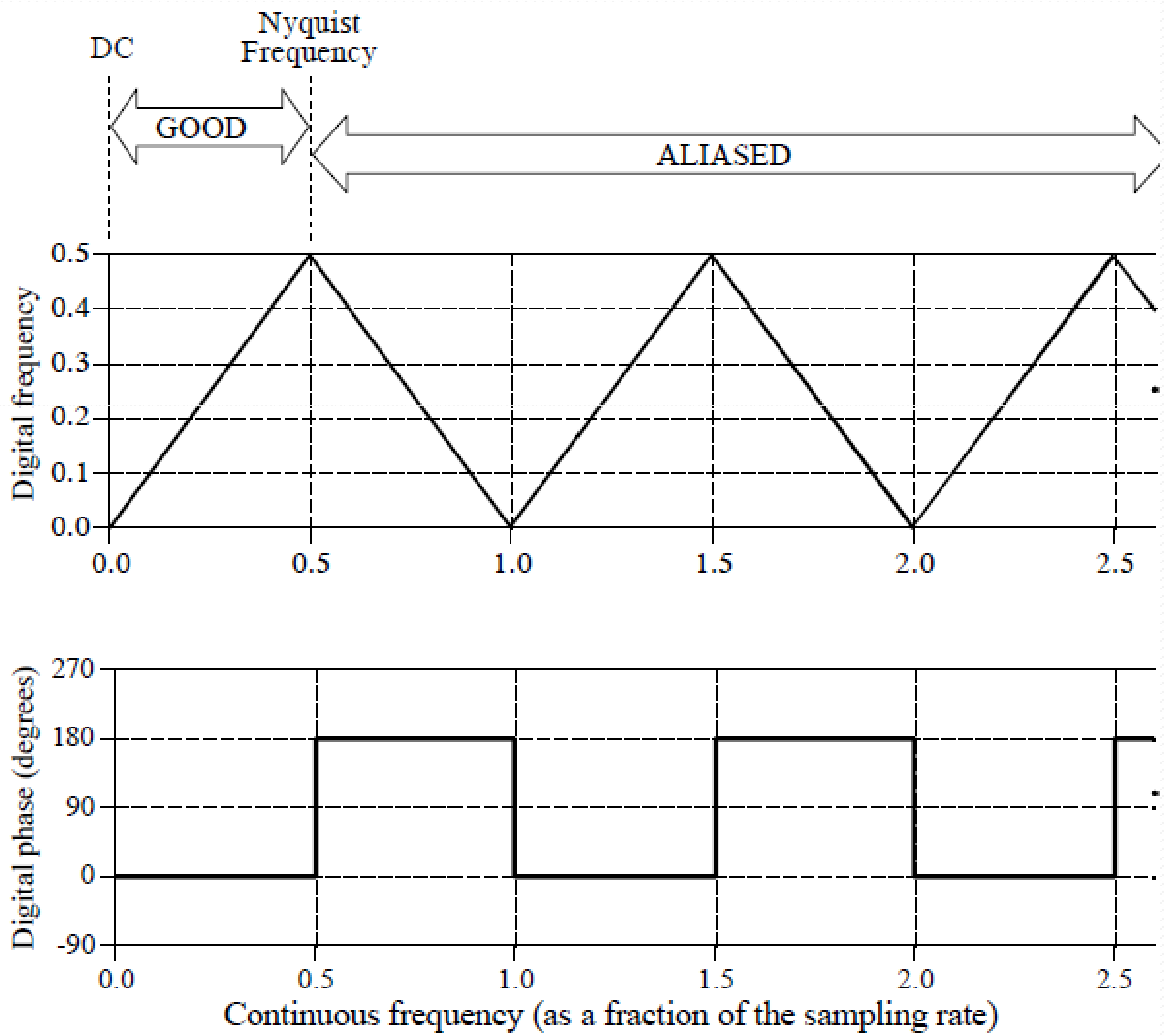
- The maximum frequency representable in the discrete signal without aliasing

$$f_n = \frac{f_s}{2}$$

Aliasing

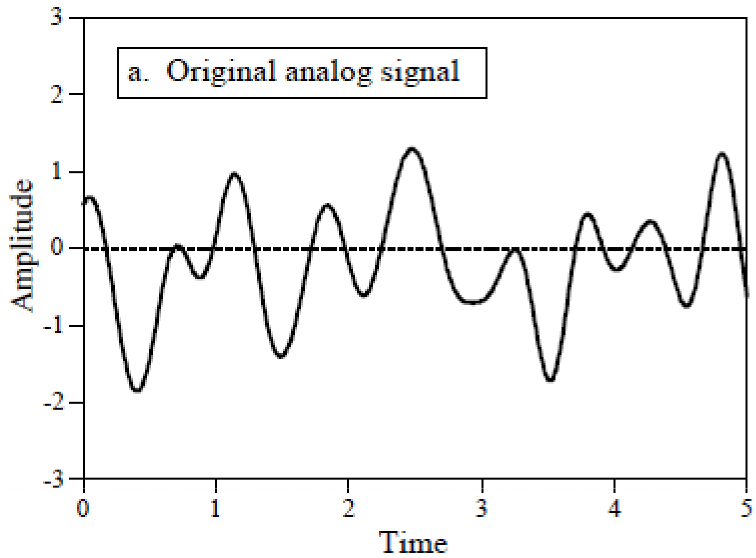
Aliasing causes information loss about both high and low frequencies

Aliasing causes a phase shift of π or zero as follows

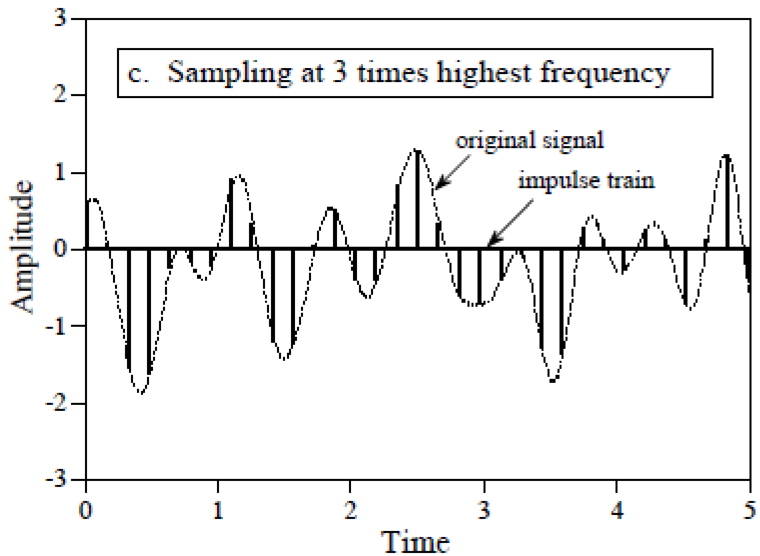


Impulse Train

- Identical information to the discrete signal

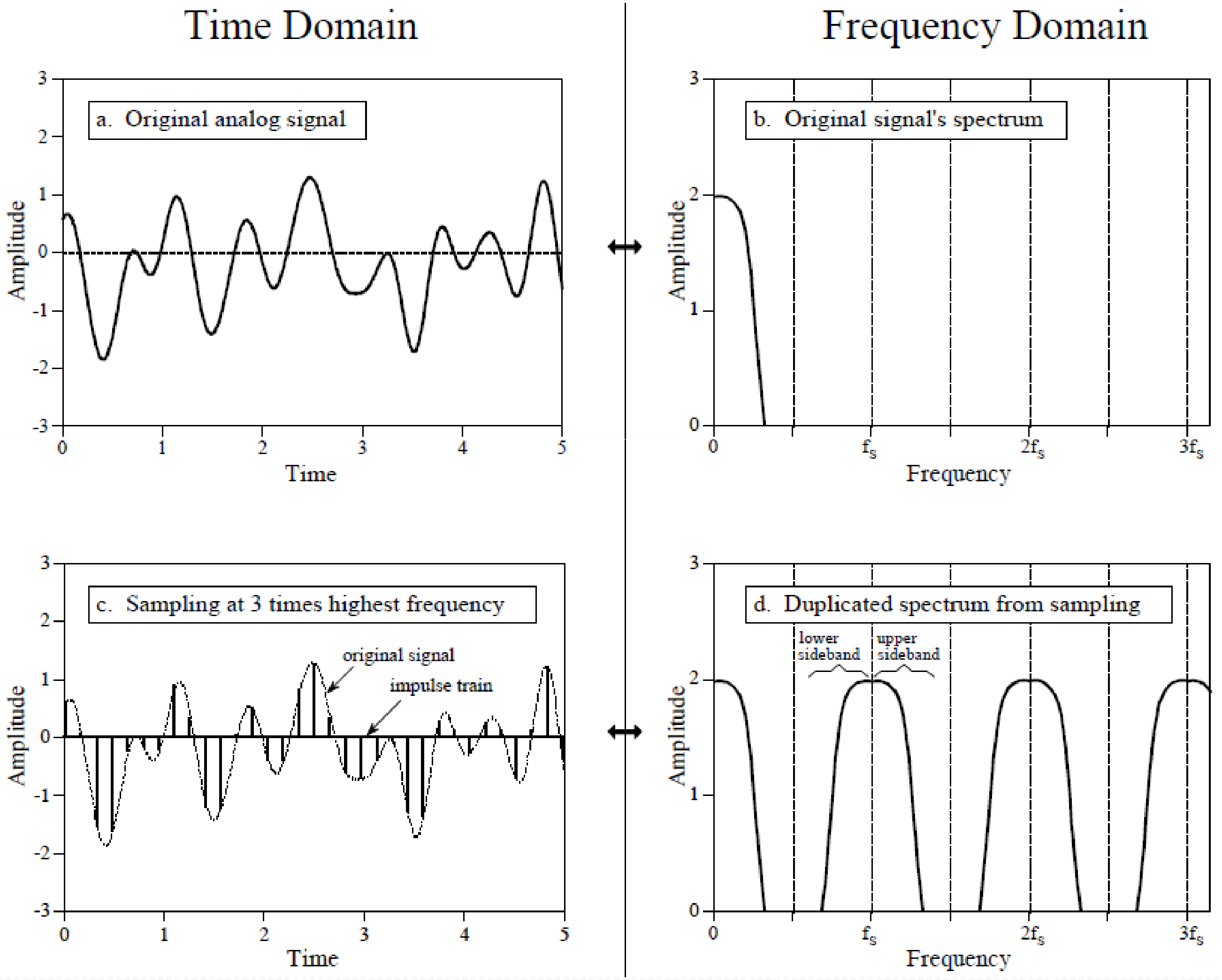


- Comparable to the continuous signal

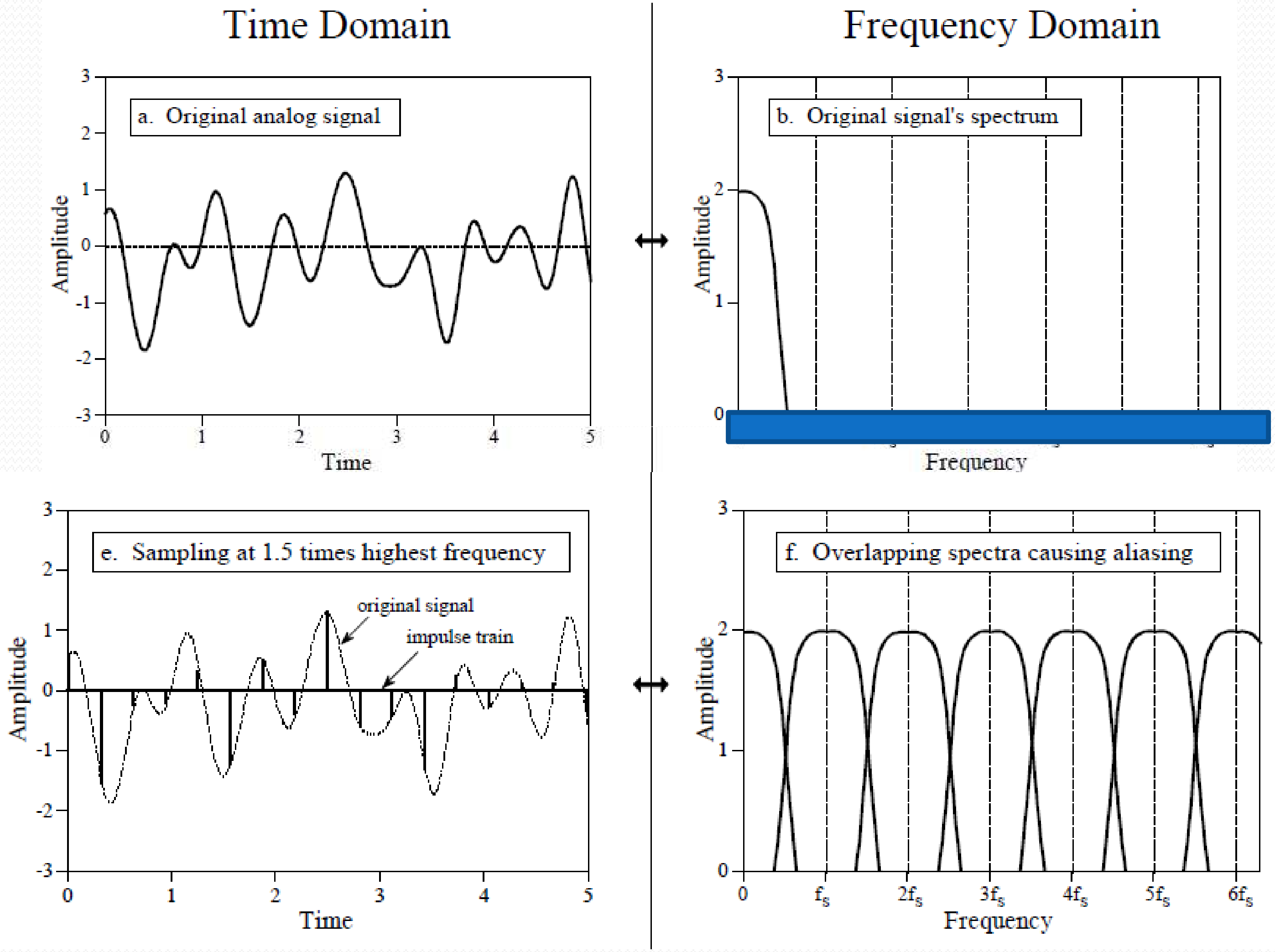


- Continuous signal can be perfectly reconstructed by passing the impulse train through a low pass filter if sampling was proper

Proper Sampling Example



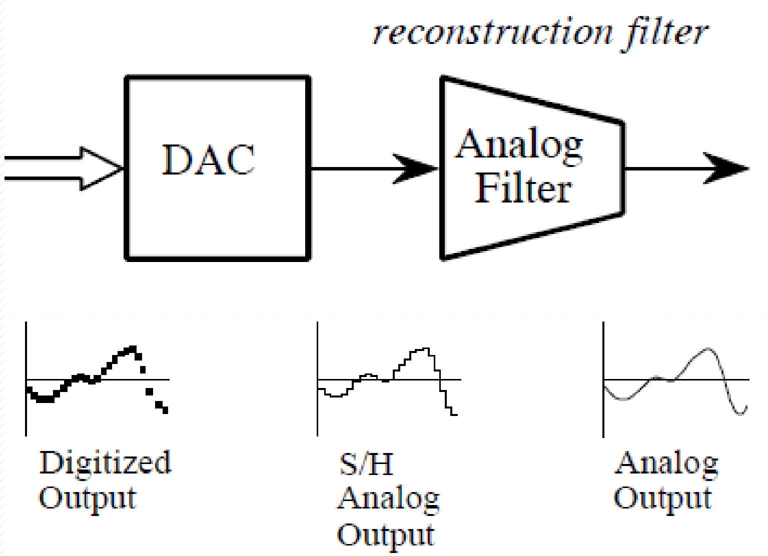
Aliasing Example

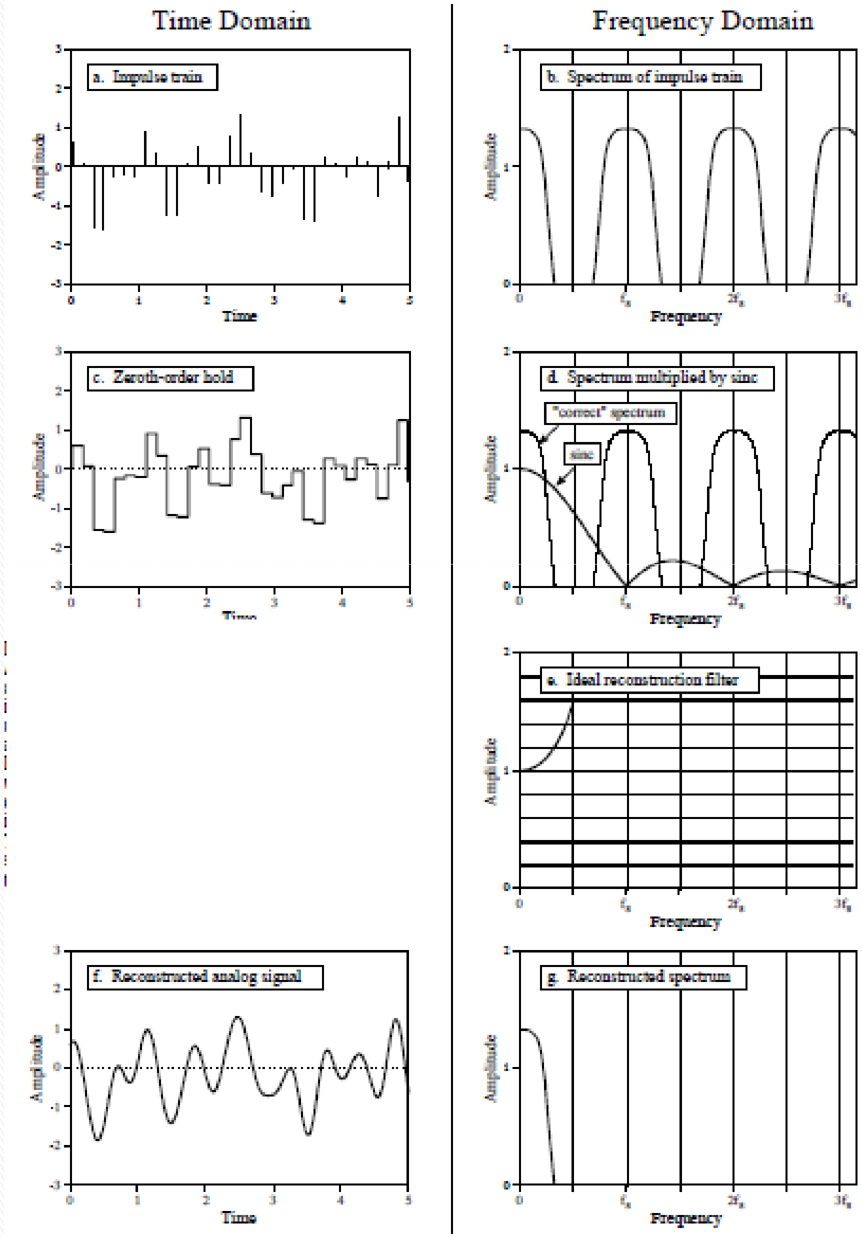


DAC

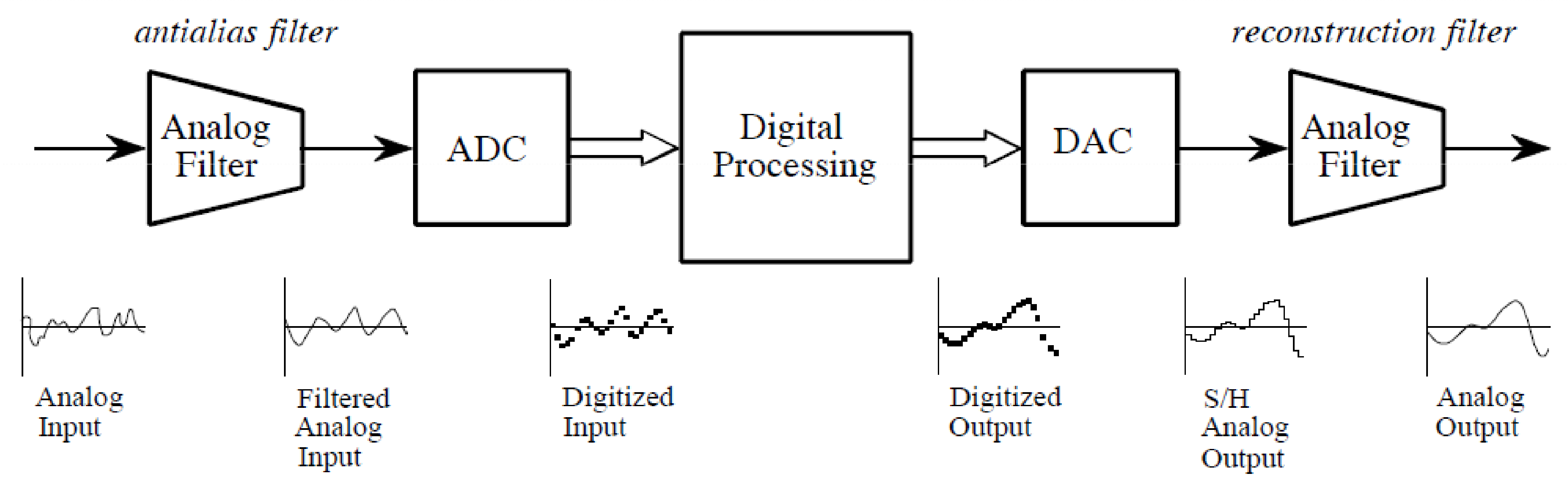
- Zeroth-order hold (generates quantized continuous signal)
  - Similar to S/H
- Reconstruction Filter
  - Removes all frequencies above half the sampling rate
  - Boost the signal by the reciprocal of the zeroth order hold effect

DAC Example


reconstruction filter
DAC
Analog Filter
Digitized Output
S/H Analog Output
Analog Output


Time Domain
Frequency Domain
a. Impulse train
b. Spectrum of impulse train
c. Zeroth-order hold
d. Spectrum multiplied by sinc
"correct" spectrum
sinc
e. Ideal reconstruction filter
f. Reconstructed analog signal
g. Reconstructed spectrum
Amplitude
Time
Frequency

Complete ADC/DAC system



SELF TEST: Why do we need an antialiasing filter even if we are not interested in signals over the Nyquest frequency?

SELF READING

- Analog Filters for Data Conversion
- Single Bit Data Conversion
- Pages 48-66