EE327 Digital Signal Processing DFT Applications

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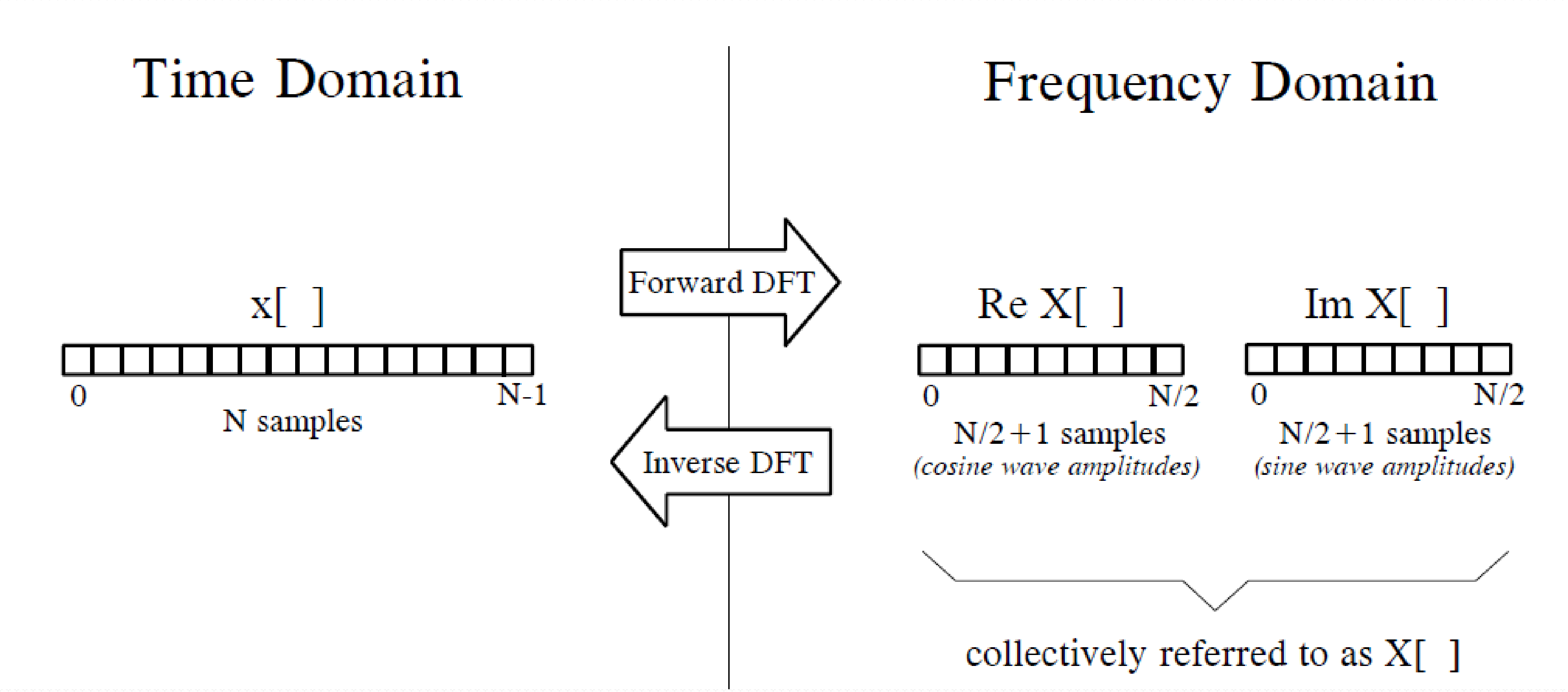
REMINDER 1: Fourier Decomposition

- Periodic Time Domain → Discrete Frequency Domain
- Discrete Time Domain → Periodic Frequency Domain

		Periodicity	
Continuity		Periodic	aperiodic
	continuous	Fourier Series FS Aperiodic Spectrum Discrete Spectrum	Fourier Transform FT Aperiodic Spectrum Continuous Spectrum
	discrete	Discrete Fourier Transform DFT Periodic Spectrum Discrete Spectrum	Discrete Time Fourier Transform DTFT Periodic Spectrum Continuous Spectrum

REMINDER 2: Discrete Fourier Transform



Usually N is a power of 2 (to use FFT)

REMINDER 3: Synthesis Equation

- From Frequency domain to Time domain

$$x[i] = \sum_{k=0}^{N/2} Re\bar{X}[k]\cos(2\pi ki/N) + \sum_{k=0}^{N/2} Im\bar{X}[k]\sin(2\pi ki/N)$$

$$Re\bar{X}[k] = \frac{ReX[k]}{N/2}$$

$$Im\bar{X}[k] = -\frac{ImX[k]}{N/2}$$

except for two special cases:

$$Re\bar{X}[0] = \frac{ReX[0]}{N}$$

$$Re\bar{X}[N/2] = \frac{ReX[N/2]}{N}$$

REMINDER 4: DFT by correlation

- Find the correlation between the basis function and the signal
- The average of this correlation is the required amplitude.

- For this to work all basis functions must have zero correlation.

- Sins and Cosines of different frequency have zero correlation

$$ReX[k] = \sum_{i=0}^{N-1} x[i] \cos(2\pi k i / N)$$

$$ImX[k] = -\sum_{i=0}^{N-1} x[i] \sin(2\pi k i / N)$$

REMINDER 5: Conversion Formulas

$$MagX[k] = \left(ReX[k]^2 + ImX[k]^2 \right)^{1/2}$$

$$PhaseX[k] = \arctan\left(\frac{ImX[k]}{ReX[k]} \right)$$

$$ReX[k] = MagX[k] \cos(PhaseX[k])$$

$$ImX[k] = MagX[k] \sin(PhaseX[k])$$

Applications of DFT

- Finding Signal's Frequency Spectrum
  - Understanding frequency contents of signals
- Finding System's Frequency Response
  - Analyzing Systems in the Frequency domain
- Intermediate Step for other operations
  - FFT convolution

Information Coding in Signals

- Information in the time domain
  - Shape
  - Examples:
    - Readings of a sensor over time
    - Stock market signals
- Information in frequency domain
  - Amplitude
  - Phase
  - Frequency
  - Examples:
    - FM radio information
    - 50Hz noise

Understanding Signal's Frequency Spectrum

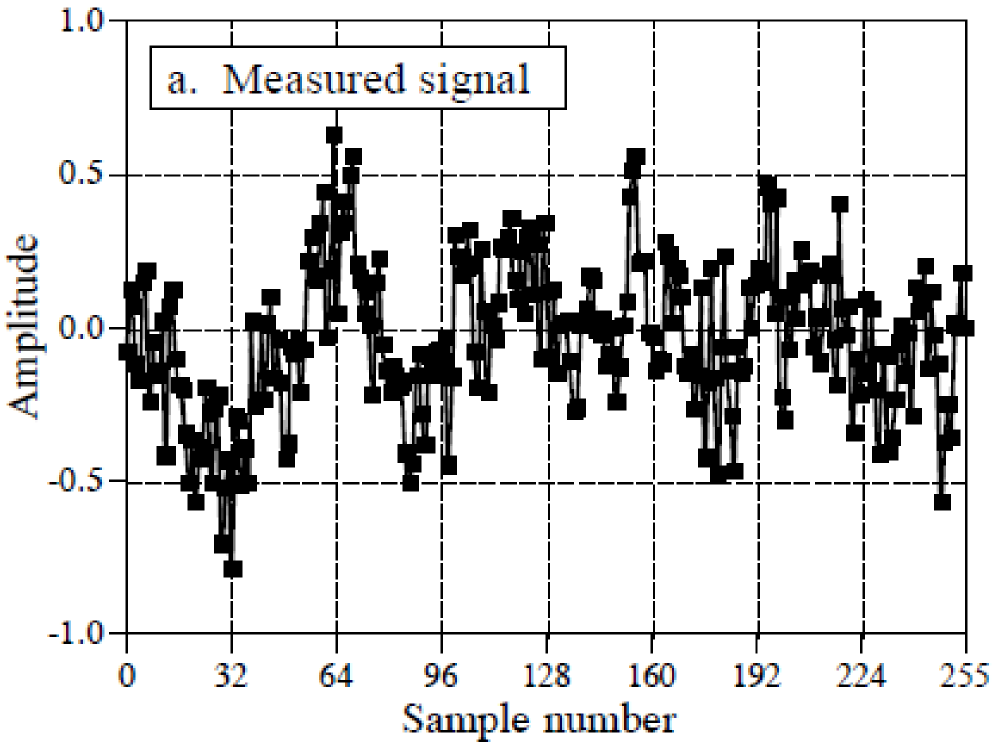
1. Collect data
2. Use DFT to convert it to frequency domain
3. Convert it to polar coordinates
4. If needed do this many times and average the results

5. Now study the spectrum

Ocean's Underwater Sounds

1. Put a microphone under water and record sound.
2. Use anti-aliasing filter to remove all frequency contents over 80 Hz.
3. Use sampling rate of 160Hz to digitize the signal
4. Collect several thousand samples of the digitized version

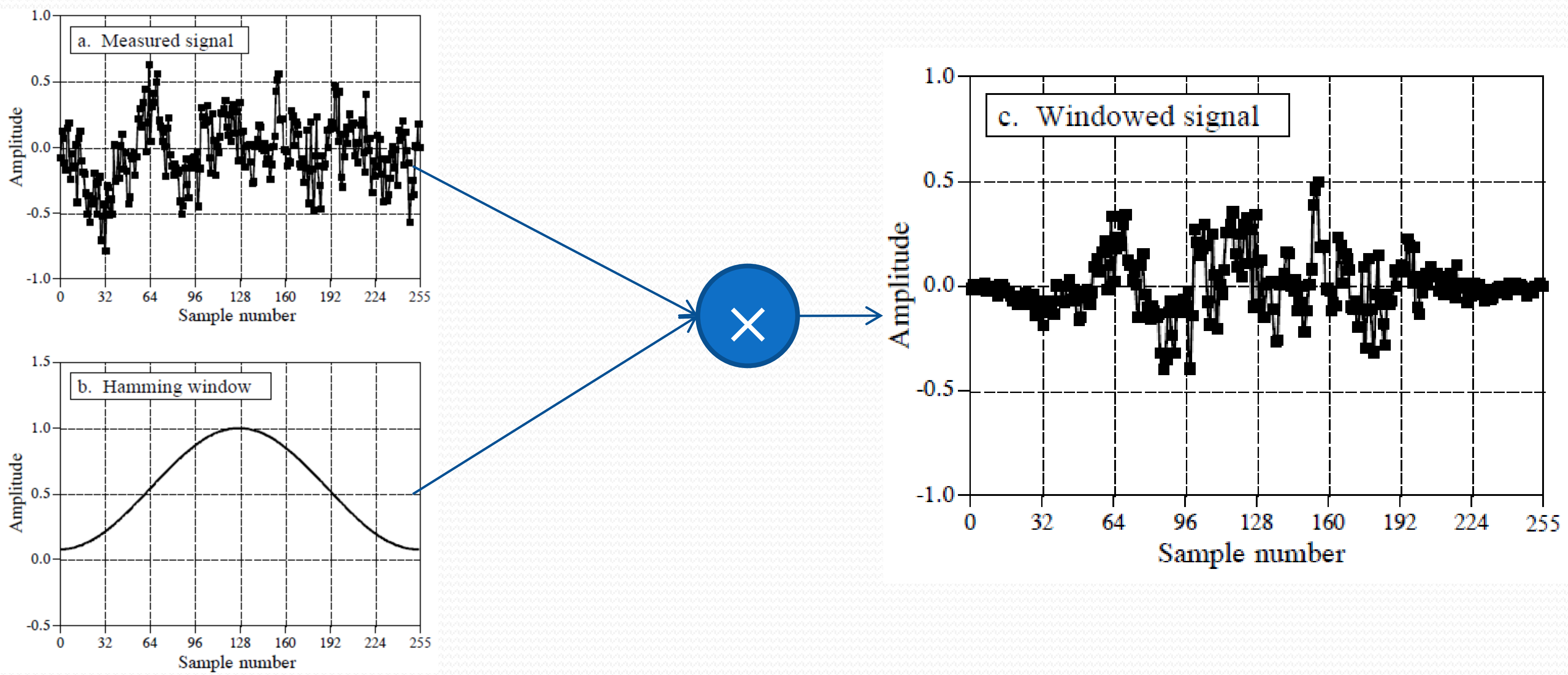
Now how does it look like?



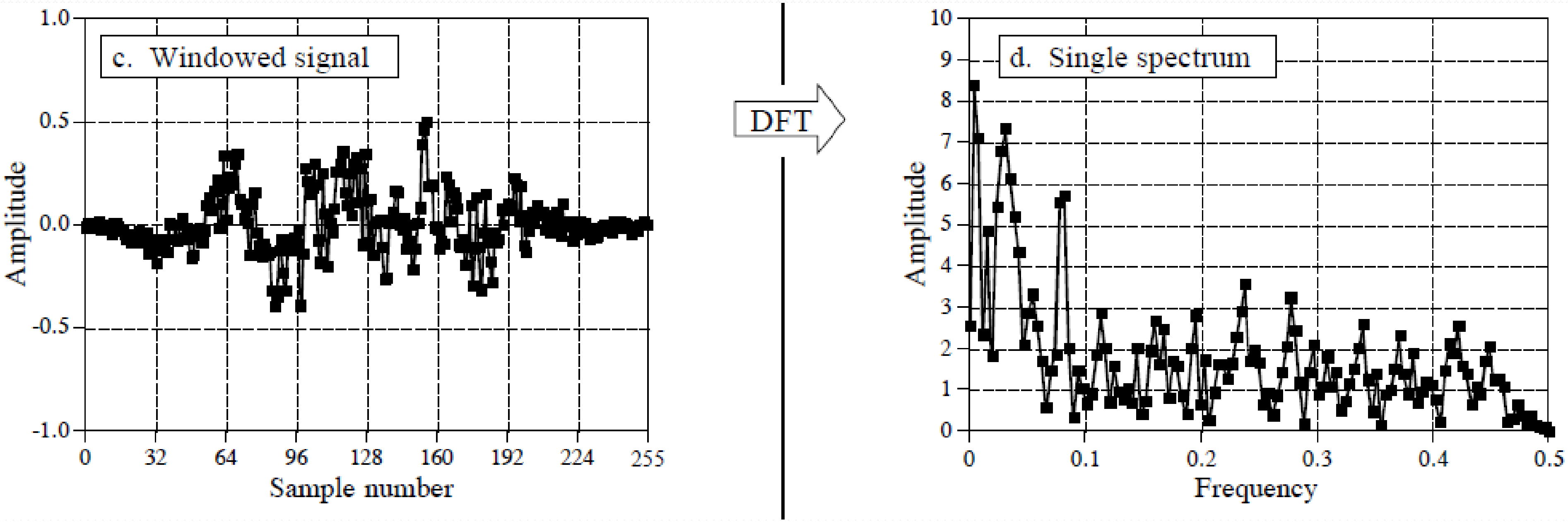
- What can you see?????
  - I can see nothing of value

Let’s prepare to the frequency domain

- Multiply the signal with a **Hamming Window**
  - Why? Just wait for few more slides



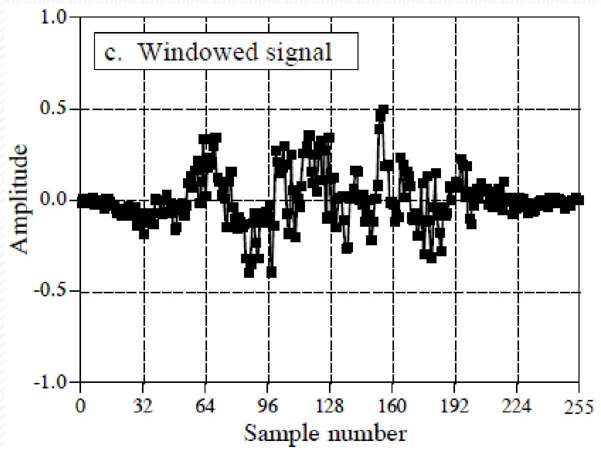
Let’s go to the frequency domain

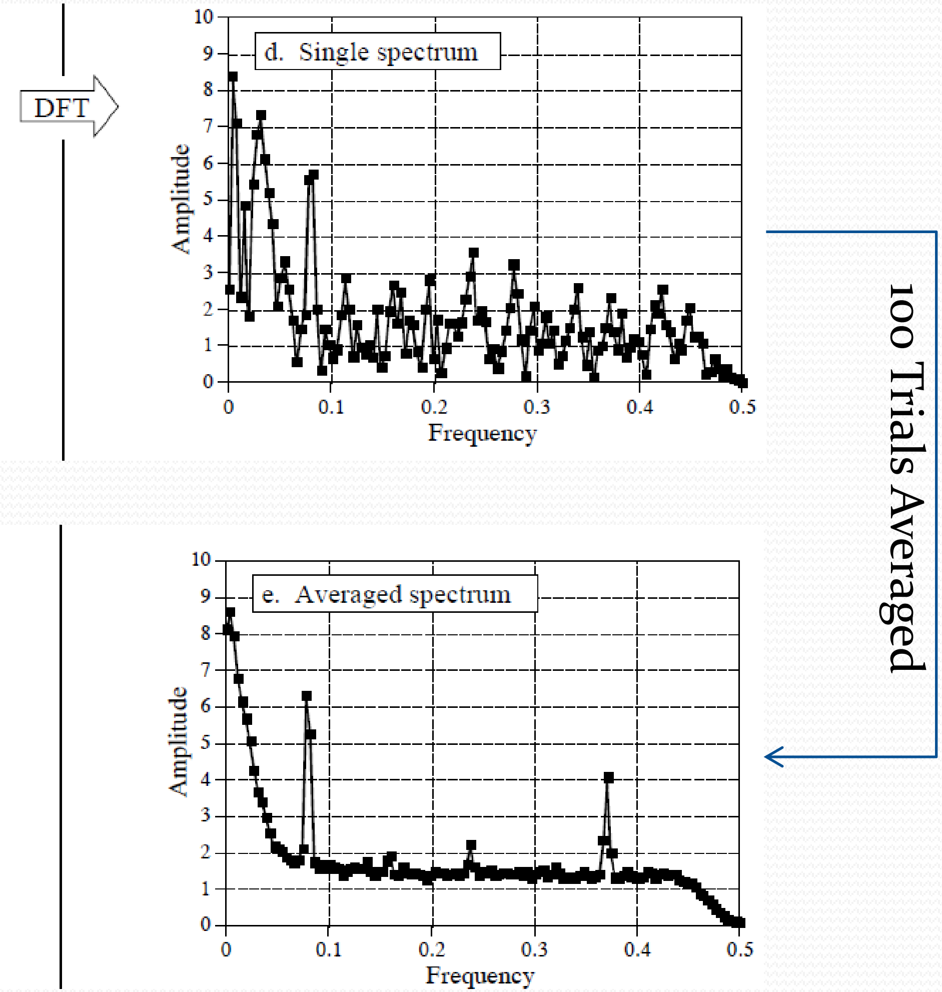


What can you see now????

Still nothing!!!

Let’s reduce the noise levels





$$Noise\ \ Level\ \ \alpha\ \ \sqrt{N.Trials}$$

Reducing noise levels (method 2)

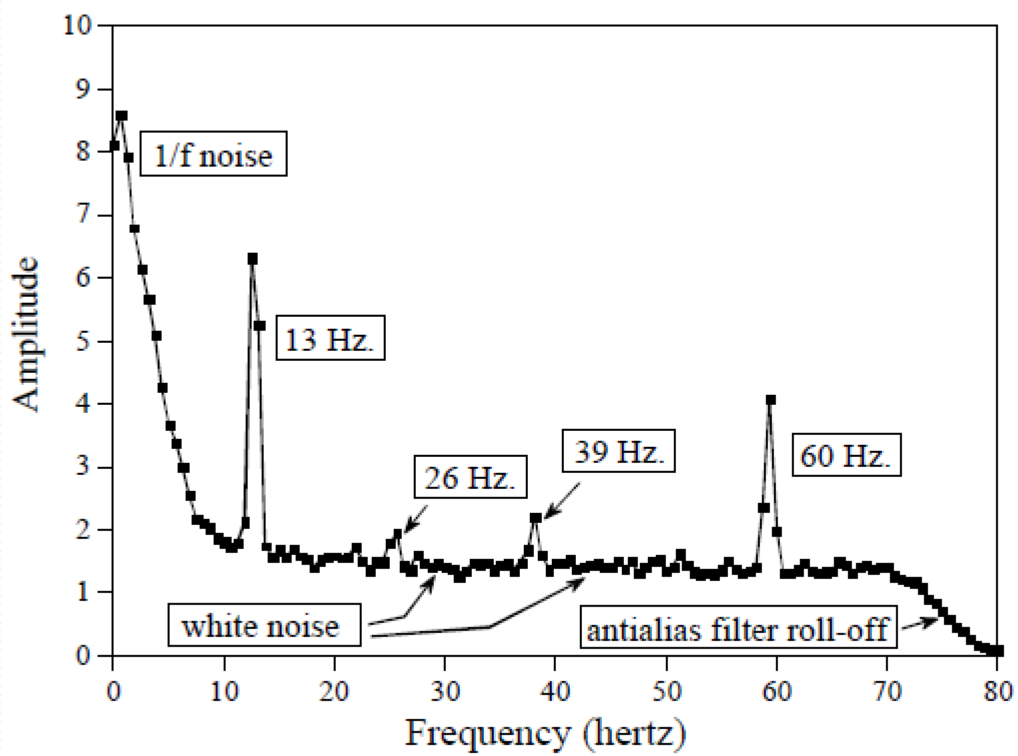
- Use a long initial time signal (e.g. 16384)
- This will give a long high resolution frequency spectrum (e.g. 8193 in each signal)
- Now average the frequency spectrum (e.g. 64 samples for each output sample)

- This gives 256 points with nearly the same noise level as in the previous method.

- *A more complex filter than averaging can be used*

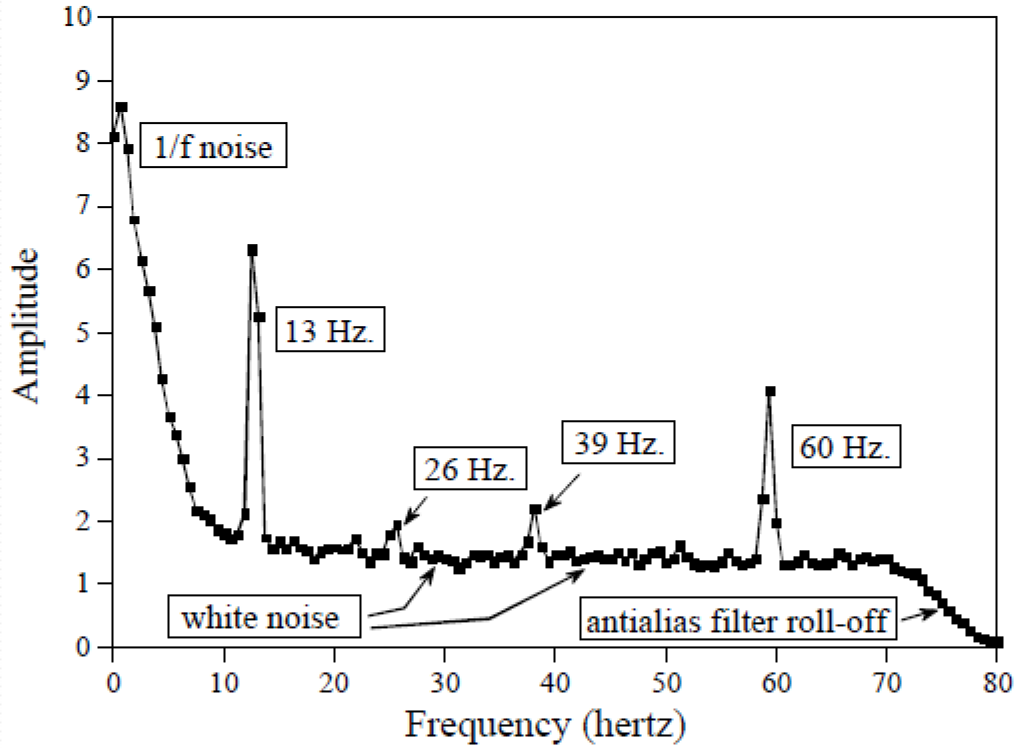
Let's study the spectrum
First, noises

- White noise
  - In all frequencies
  - Uncorrelated with signal
  - Example: water spray
- Antialias roll-off
  - No perfect antialias filter!!
- 1/f noise ()
  - A mystery everywhere!!
  - Between 1 and 100Hz usually



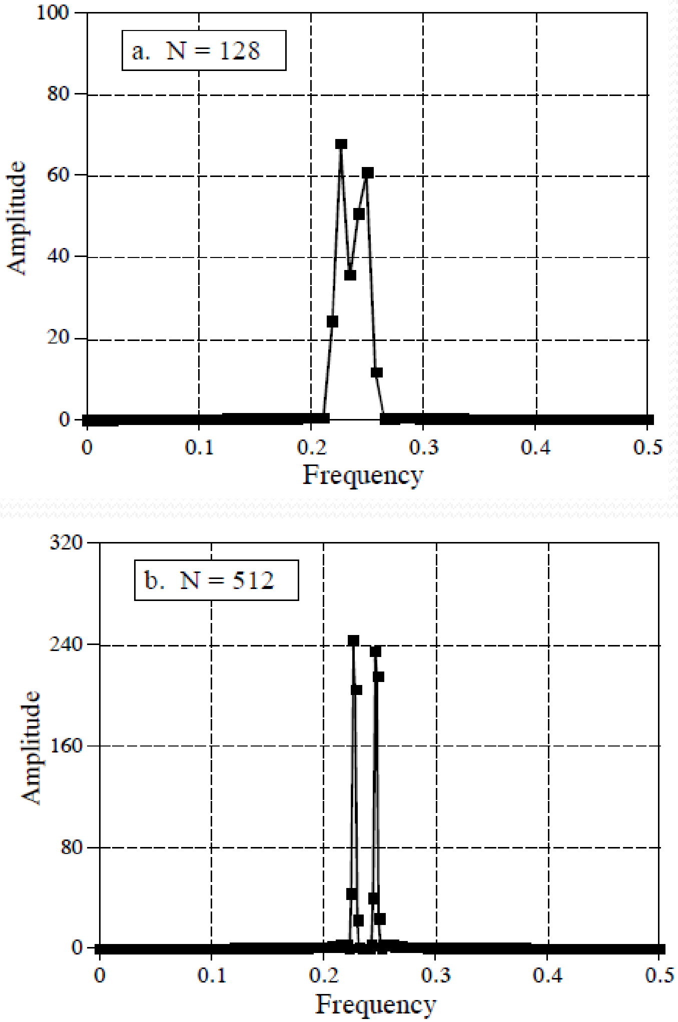
Let's study the spectrum
Second, useful signals

- 13, 26, 39 Hz
  - Nonsinusoidal periodic signal.
  - Fundamental is 13
  - Harmonics are 26 and 39
  - May be a submarine!!!
- 60Hz
  - Noise from AC power sources



Separating Features

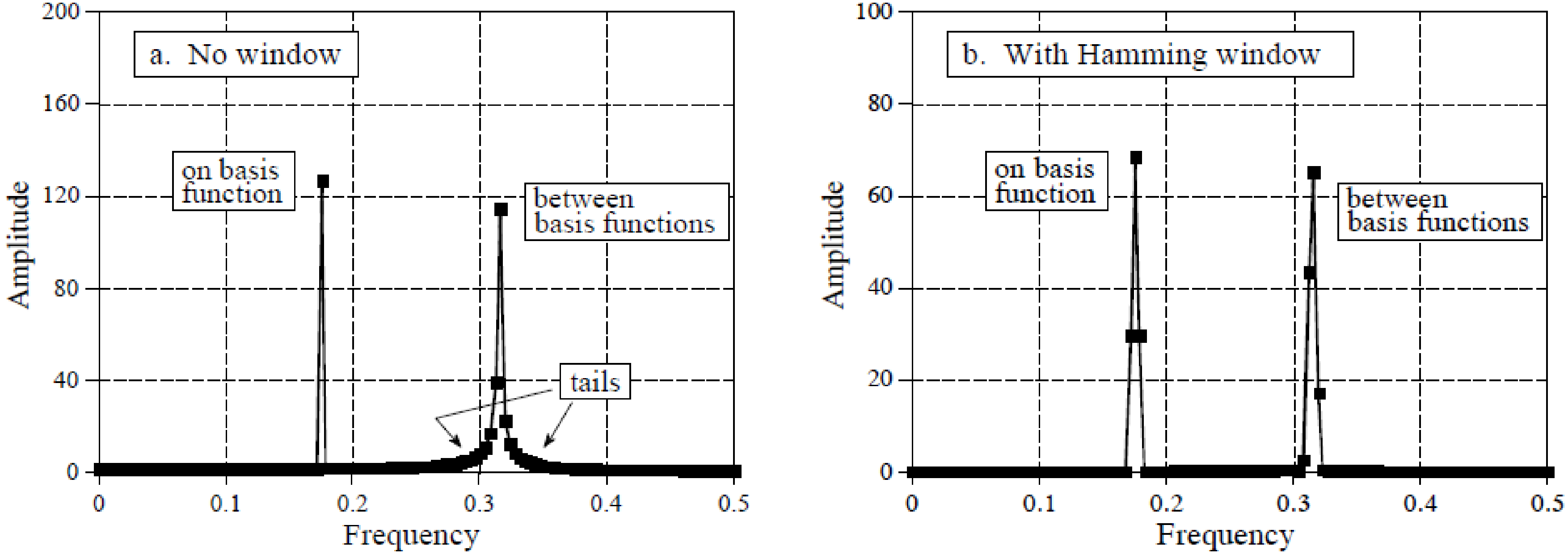
- Why features merge?
  1. Small size of the DFT
     - Solution: Increase DFT size
  2. Small input size
     - Solution: collect more time domain information



Why they are different?

- The length of ReX and ImX are N/2+1 where N is the size of time domain signal
- Yet, size of time domain signal $\neq$ DFT size
- How?
  - To increase DFT size, extend the signal with zeros to whatever N you want
- Why it works?
  - DFT assumes that the signal is periodic … extending with zeros CHANGES the signal

What about frequencies in between

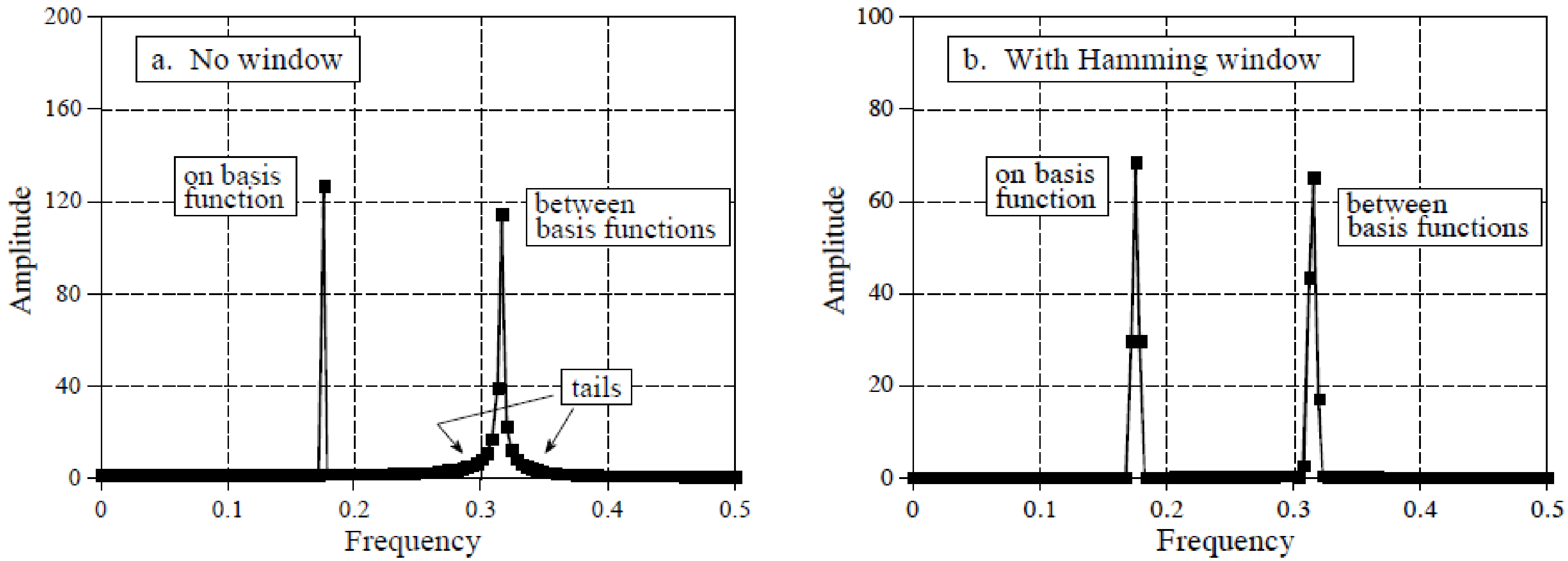


- Matching frequencies appear as single points
- Frequencies in between basis functions appear with tails

- To reduce tails: Multiply with a hamming window BEFORE DFT

Effects of Hamming Window

- GOOD: Reduces the tails
- GOOD: Makes on basis and between basis frequencies look the same
- BAD: Increases spike width (specially for on basis)



Applications of DFT

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  - ~~Understanding frequency contents of signals~~

- Finding System's Frequency Response
  - Analyzing Systems in the Frequency domain

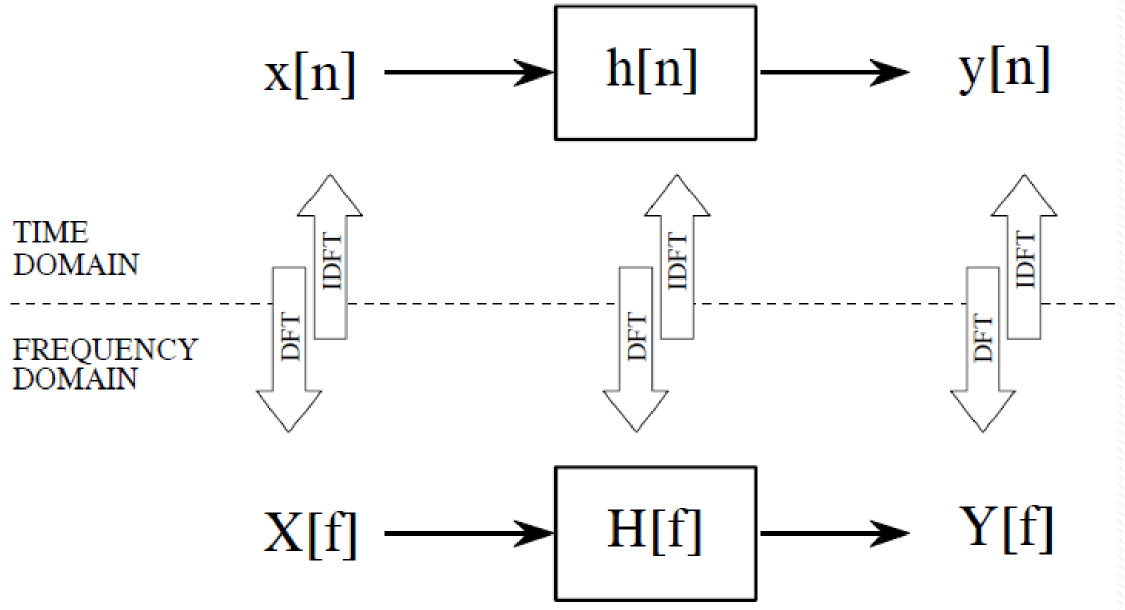
- Intermediate Step for other operations
  - FFT convolution

System's Frequency Response

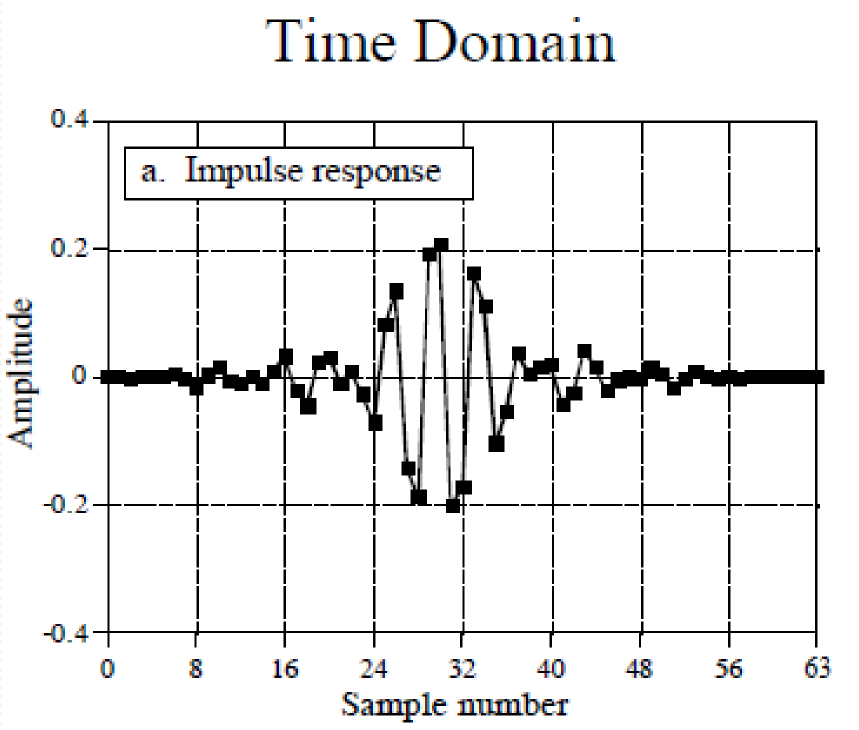
- Frequency response of a system H[*f*]:
  - Complete description of how it changes the amplitude and phase of input sinusoidals in the output.
- System's frequency response = Fourier Transform of its impulse response
- h[t] $\xrightarrow{\text{FT}}$ H[*f*]

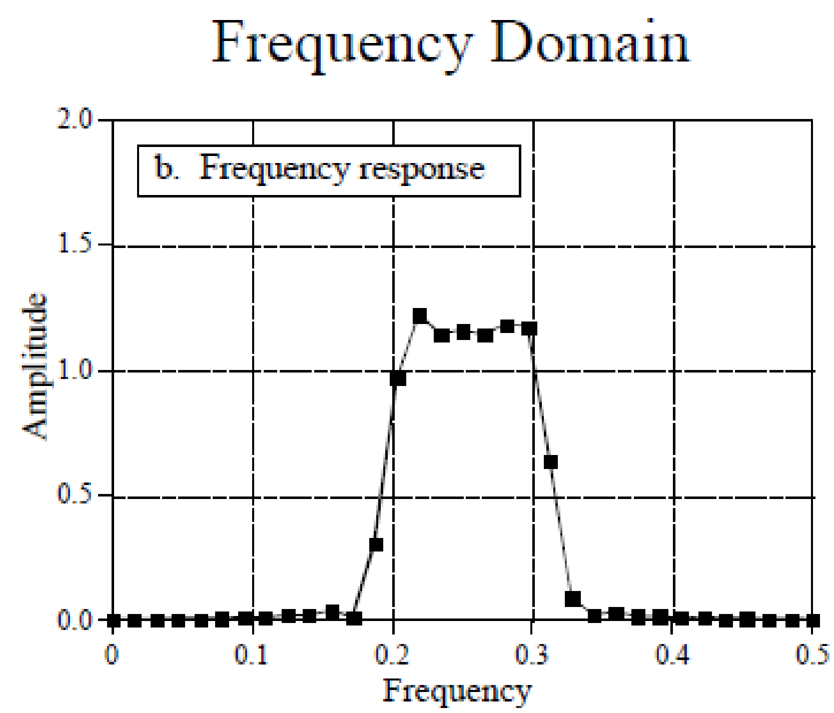
Convolution

- In time domain
  - y[n]=x[n]*h[n]   (convolution)
- In frequency domain
  - Y[*f*]=X[*f*]×H[*f*]   (multiplication)



Looking at the frequency response

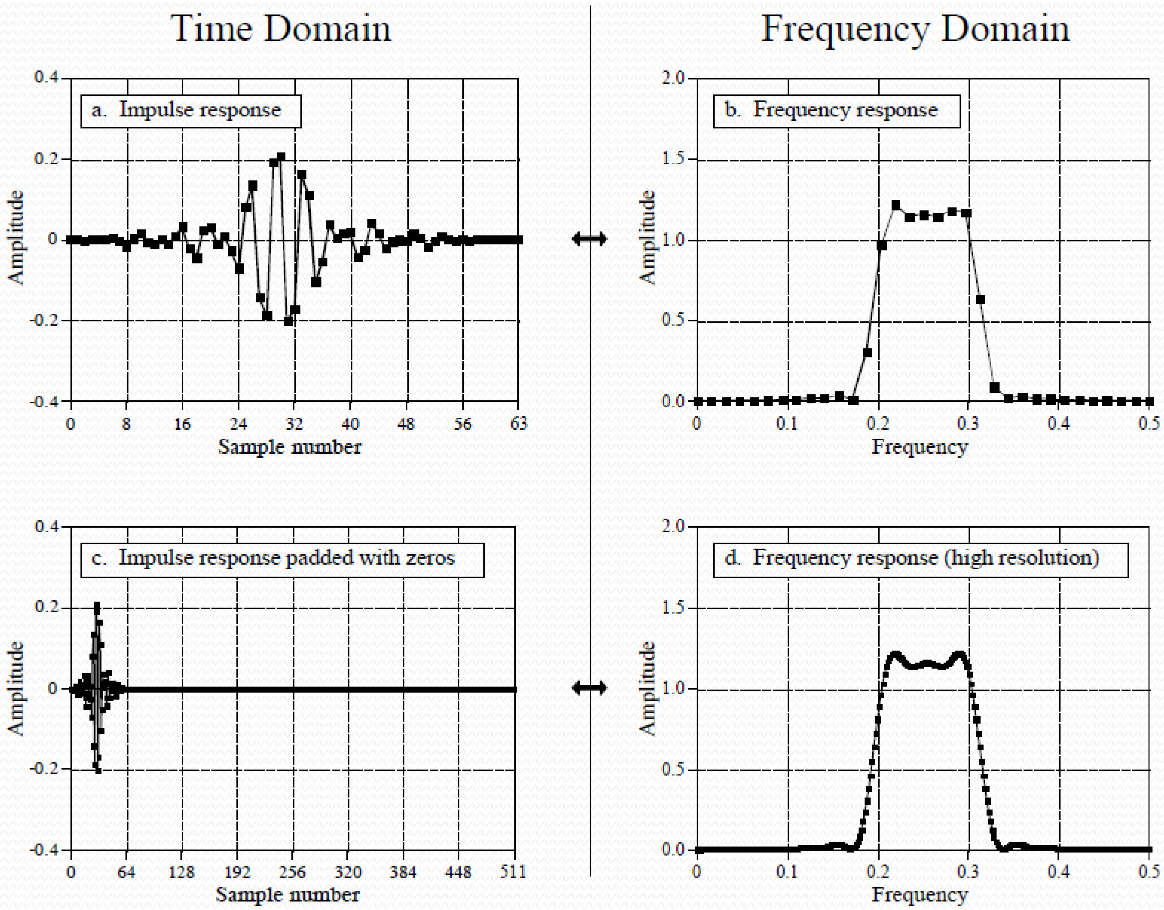




What does this system do?????????????????

Improving Frequency domain resolution

- Just pad with zeros.
- How much can you pad?
  - To infinity
- If you pad to infinity:
  - Time domain becomes aperiodic
  - Frequency domain becomes continuous
  - DTF becomes DTFT



This means that DFT is the sampling of DTFT and sampling rate is determined by N

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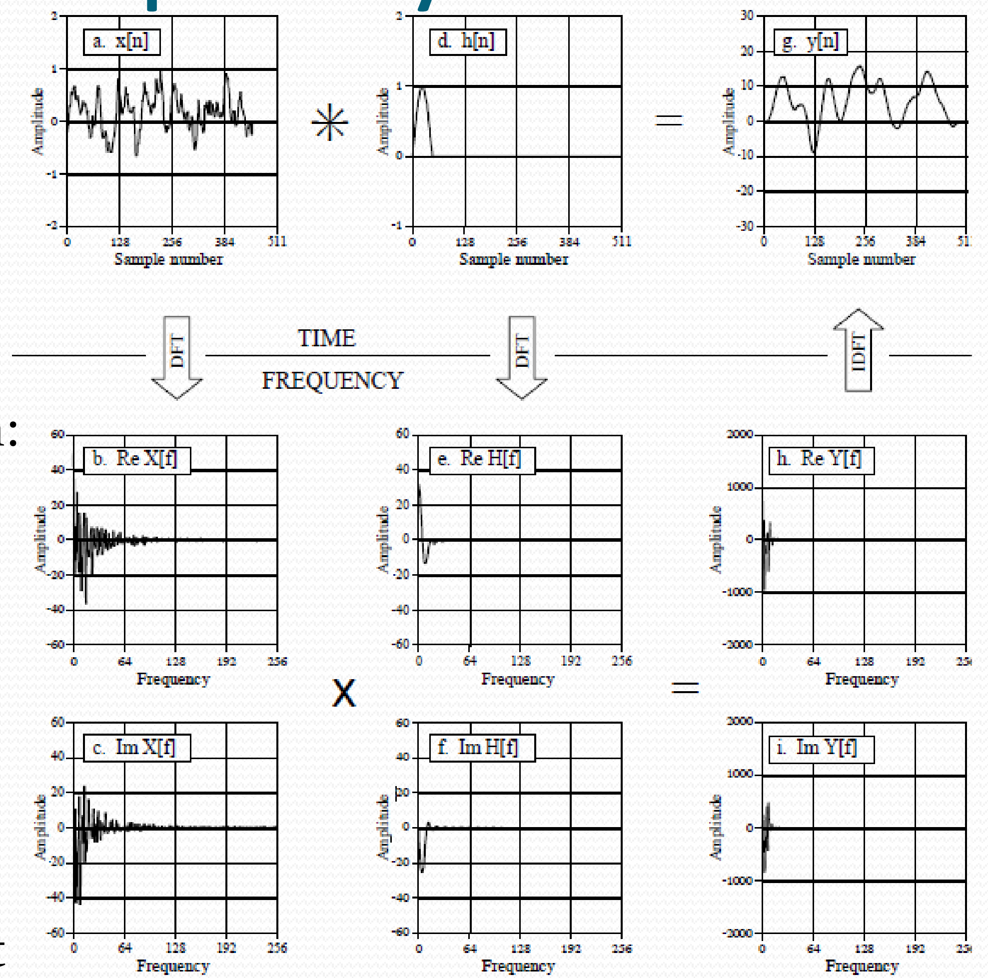
Convolution via Frequency domain

- c[n]= a[n]*b[n]

1. Pad both signals to N+M-1 points by adding zeros
2. Convert both to frequency domain:
   - MagA[f], Mag[f]
   - PhaseB[f],Phase[f]
3. Multiply in frequency domain:
   - MagC[f]=MagA[f]×MagB[f]
   - PhaseC[f]=PhaseA[f]+PhaseB[f]
4. Convert C[f] to time domain to get c[n] EXACTLY



Why on earth should we do that?

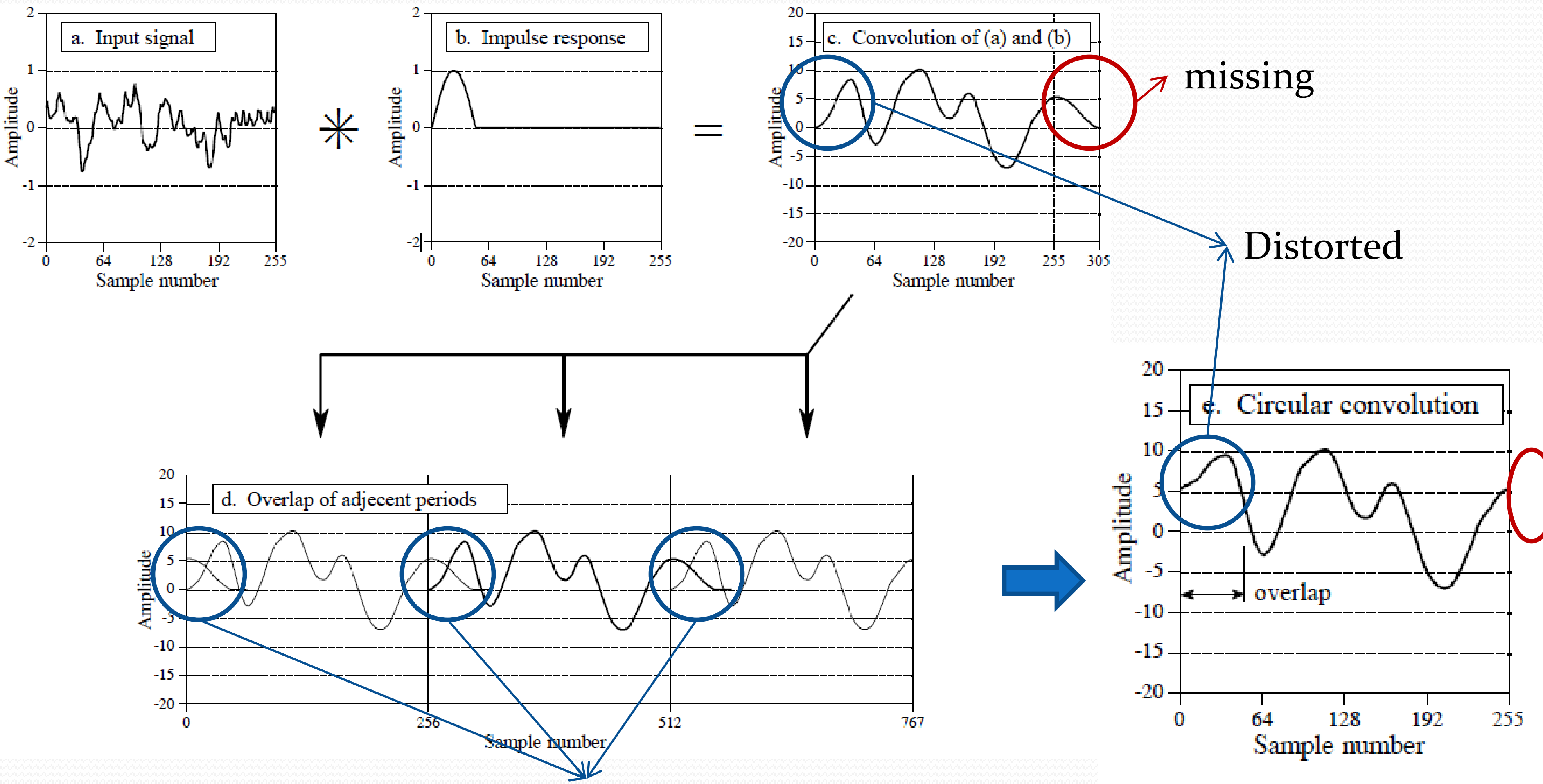
- Deconvolution is very difficult
  - Given output and impulse response finding input is difficult in time domain
  - In frequency domain it is a DIVISION

- Convolution is slow
  - DFT calculated using FFT method can give orders of magnitude improvement in speed for large signals.

Size of inputs

- Convolution of N point signal with M point signal gives:
  - N+M-1 points output signal
- To perform Convolution via frequency domain we need:
  - Pad first signal with M-1 points all zeros
  - Pad second signal with N-1 points all zeros
- This makes the final output N+M-1 points as needed
- If you added more zeros it will only add zeros to the final ouput

Circular Convolution Problem



Cause of Distortion: DFT assumes the signal is periodic

How to solve Circular convolution

- Convolution of N point signal with M point signal gives:
  - N+M-1 points output signal
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Multiplication in frequency domain

$$MagY[f] = MagX[f] \times MagH[f]$$

$$PhaseY[f] = PhaseX[f] + PhaseH[f]$$

Division in frequency domain

$$MagH[f] = MagY[f] / MagX[f]$$

$$PhaseH[f] = PhaseY[f] - PhaseX[f]$$

Multiplication in frequency domain

$$ReY[f] = ReX[f]\,ReH[f] - ImX[f]\,ImH[f]$$

$$ImY[f] = ImX[f]\,ReH[f] + ReX[f]\,ImH[f]$$

Division in frequency domain

$$ReH[f] = \frac{ReY[f]\,ReX[f] + ImY[f]\,ImX[f]}{ReX[f]^2 + ImX[f]^2}$$

$$ImH[f] = \frac{ImY[f]\,ReX[f] - ReY[f]\,ImX[f]}{ReX[f]^2 + ImX[f]^2}$$