IT 422 Network Security
Public Key Cryptography

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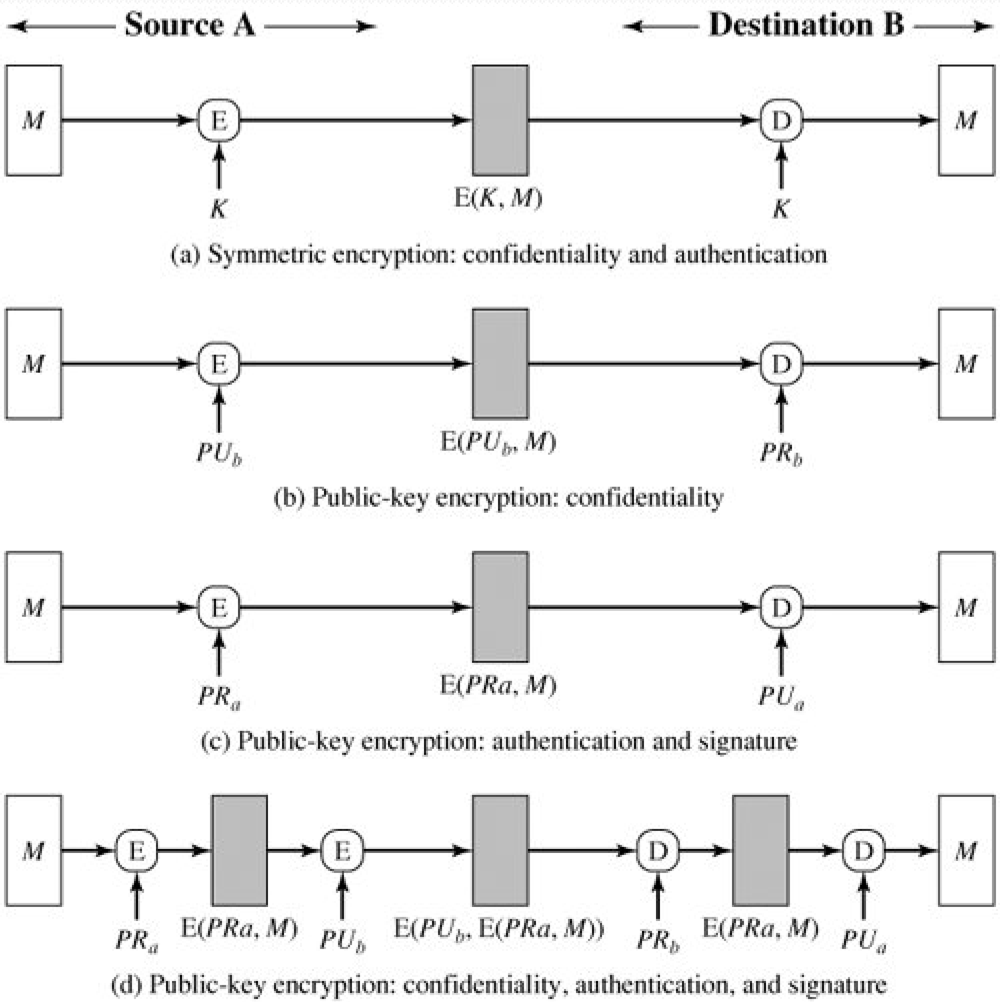
REMINDER 1: Different Uses of Encryption



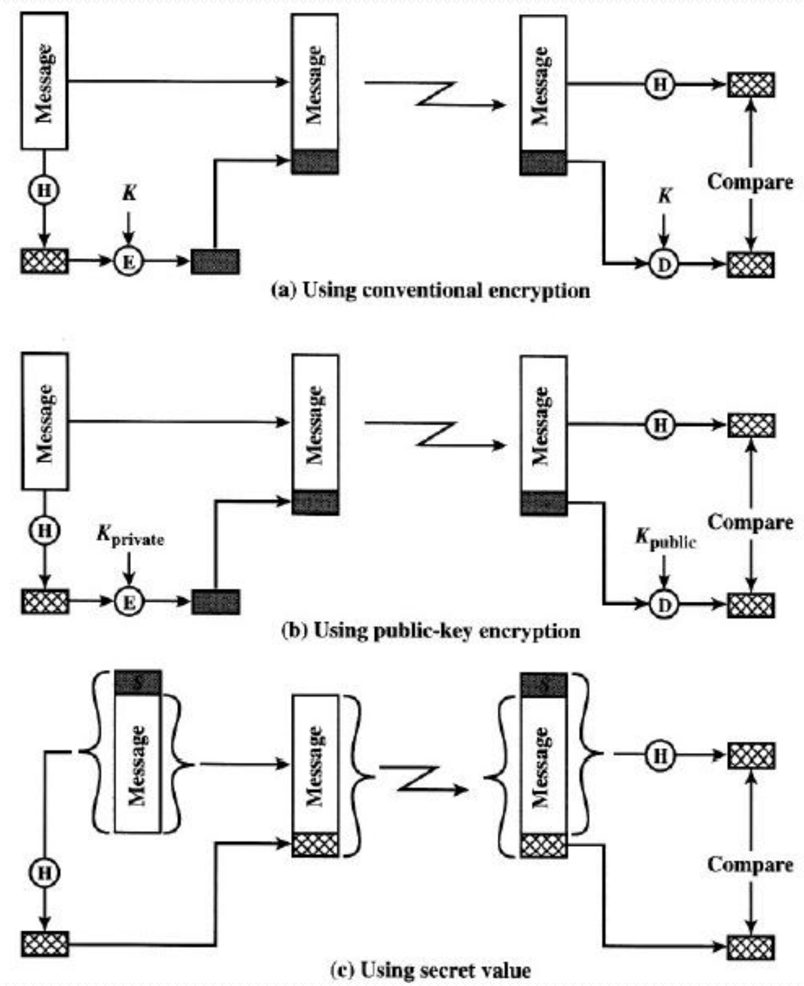
REMINDER: One Way Hash Functions

a) Only we know *k*
   - *Most conventional*

b) Uses Public Keys only
   - *Offers Nonrepudiation*
   - *No key distribution*

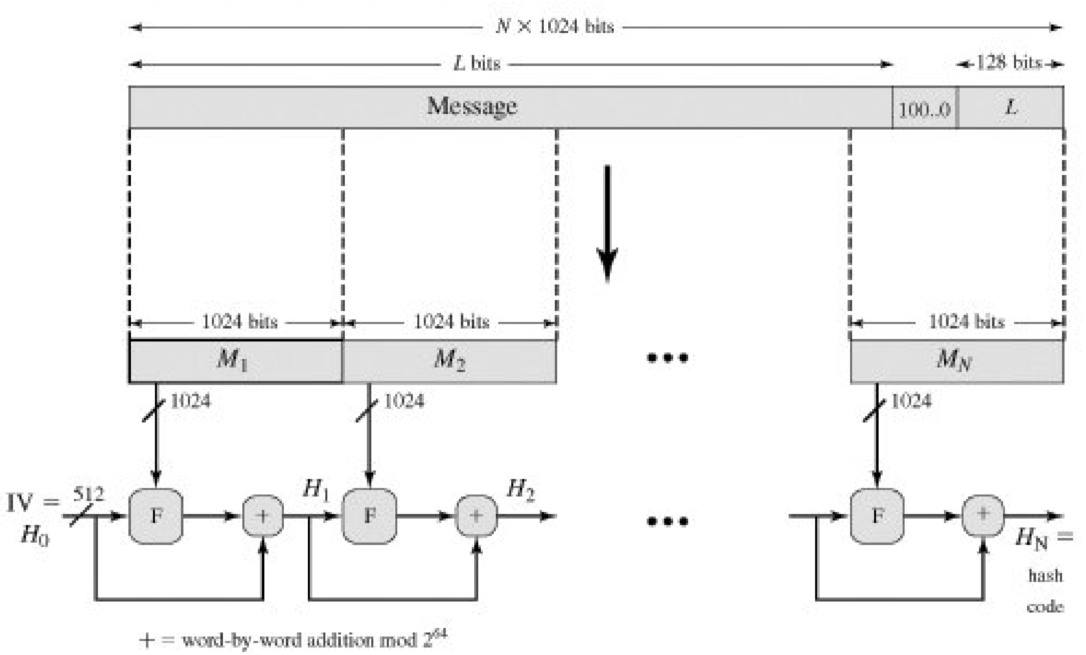
c) Only we know the secret
   - *No encryption*
   - *Used in HMAC adopted by IP security*

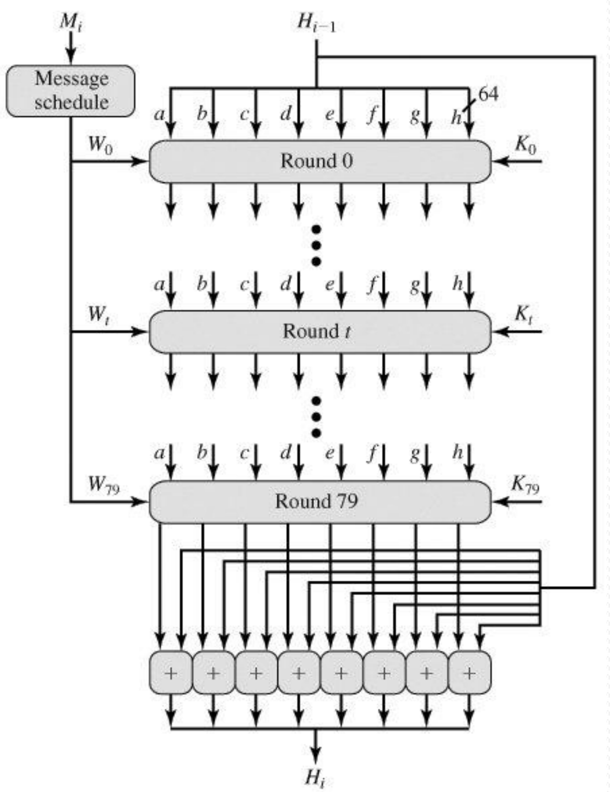
- *Why No Encryption?*
  1. *Encryption is slow*
  2. *Encryption is expensive*
  3. *Encryption is optimized for large*
  4. *Patents & export control*



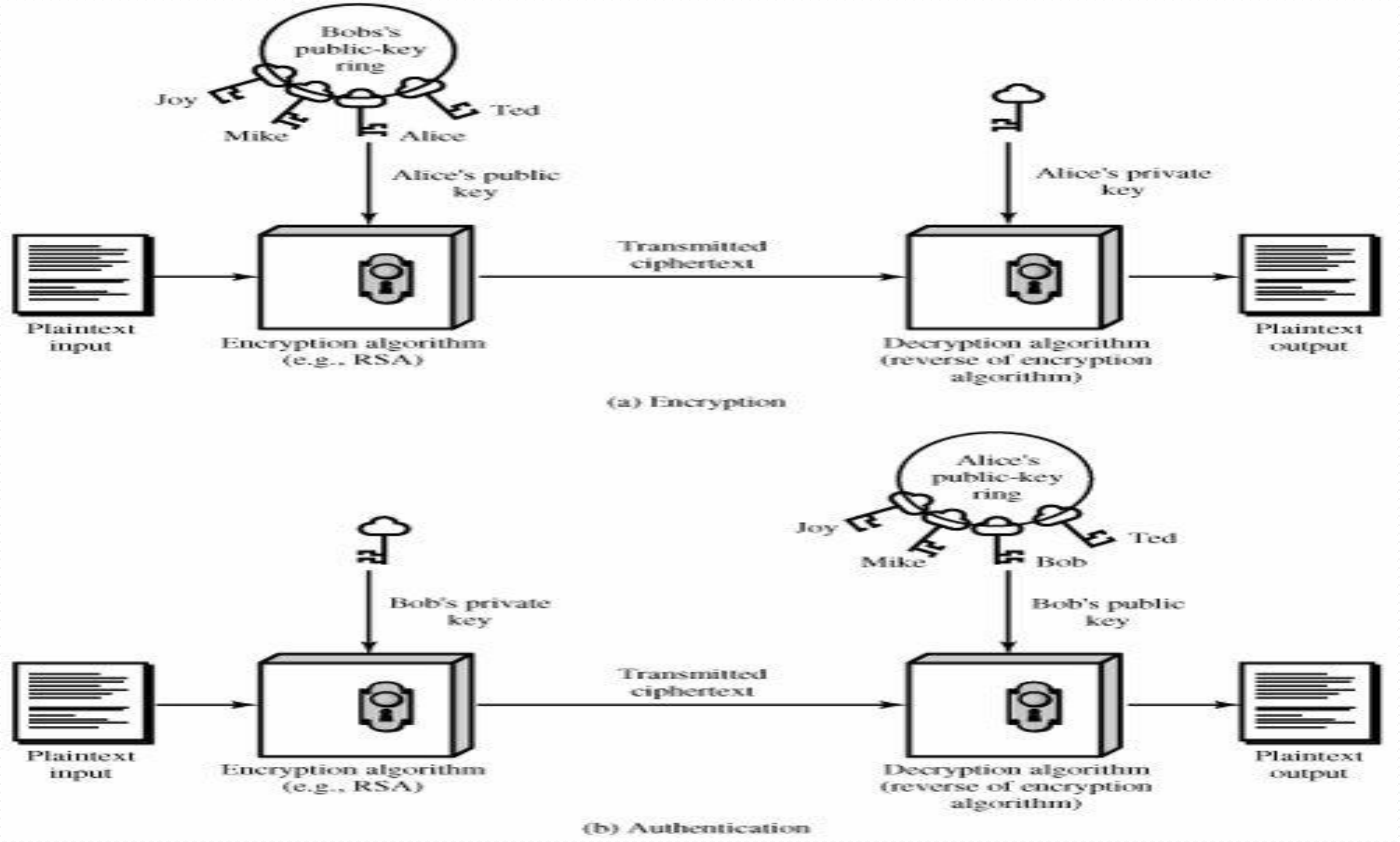
REMINDER 3: Modern Hash Functions

- SHA-1 (self read the algorithm)
  - Maximum input is 2^{64}
  - Digest size = 160 bits
  - Block size is 512 or 1024 bits





Public Key Encryption



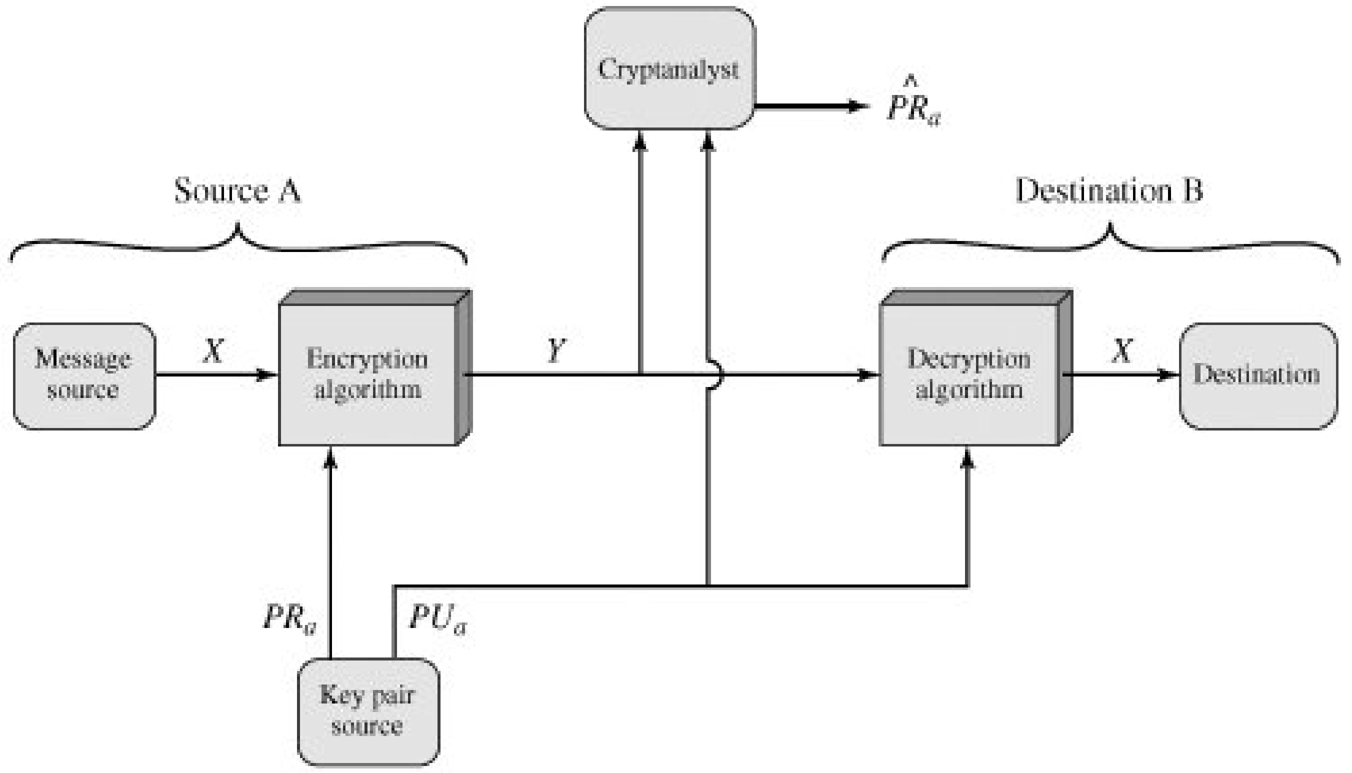
Public vs. Shared Key

Conventional Encryption	Public-Key Encryption
Needed to Work: 1. The same algorithm with the same key is used for encryption and decryption. 2. The sender and receiver must share the algorithm and the key.	**Needed to Work:** 1. One algorithm is used for encryption and decryption with a pair of keys, one for encryption and one for decryption. 2. The sender and receiver must each have one of the matched pair of keys (not the same one).
Needed for Security: 1. The key must be kept secret. 2. It must be impossible or at least impractical to decipher a message if no other information is available. 3. Knowledge of the algorithm plus samples of ciphertext must be insufficient to determine the key.	**Needed for Security:** 1. One of the two keys must be kept secret. 2. It must be impossible or at least impractical to decipher a message if no other information is available. 3. Knowledge of the algorithm plus one of the keys plus samples of ciphertext must be insufficient to determine the other key.

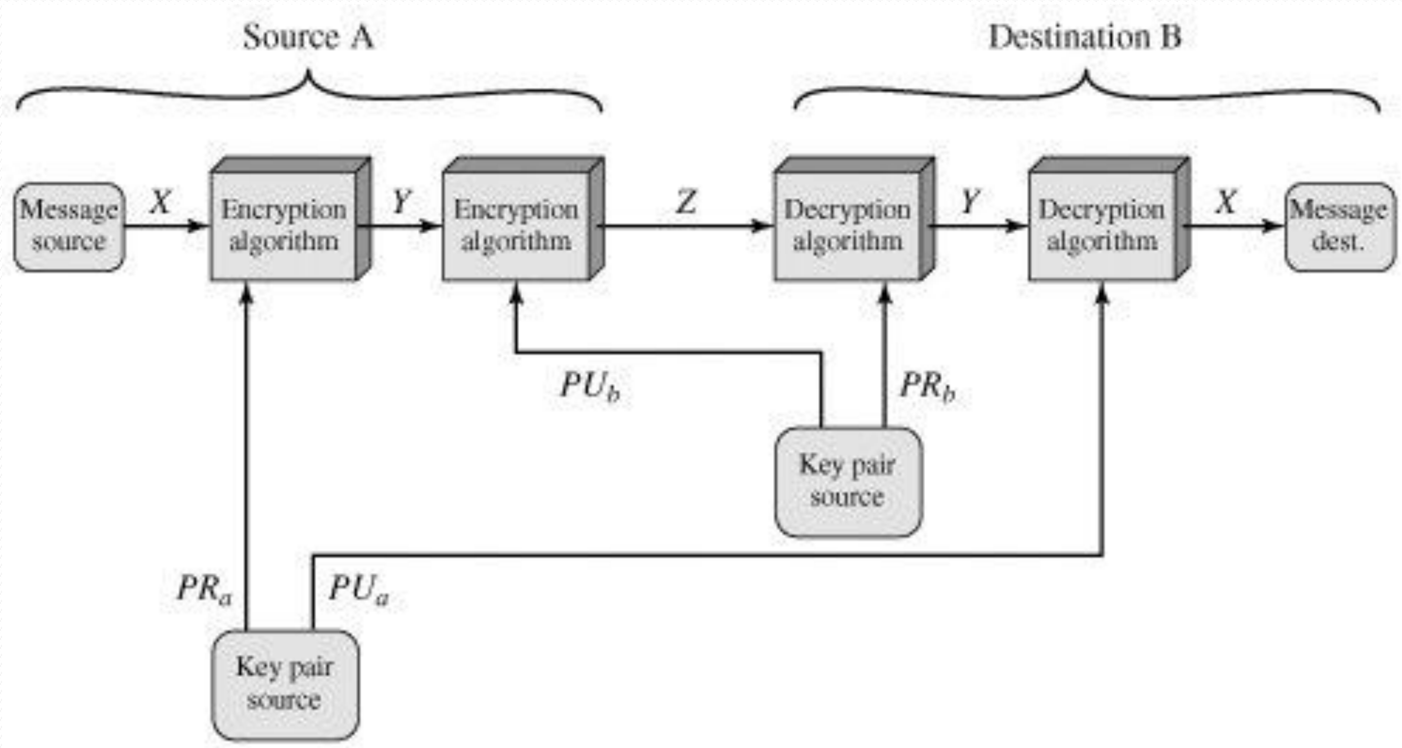
Uses of Public Key Encryption

- Encryption/Decryption
- Digital Signature
- Shared-Key Exchange

Public Key for Authentication



Public Key for Confident. + Auth.



Applications of Public Key Systems

Algorithm	Encryption/Decryption	Digital Signature	Key Exchange
RSA	Yes	Yes	Yes
Elliptic Curve	Yes	Yes	Yes
Diffie-Hellman	No	No	Yes
DSS	No	Yes	No

Requirements of Public Key Systems

- Easy to generate key pairs
- Easy to encrypt and decrypt
- Knowing the public key we cannot guess the private key
- Knowing a cipher and the public key we cannot get the plain text

- [Optional] the two keys can be applied in either order

RSA

- Developed in 1977
- By
  - Ron Rivest
  - Adi Shamir
  - Len Adelman
- Plain and ciphertexts are numbers between 0 and $2^n - 1$ (usually n=1024)
- General Purpose Public Key system
- Depends on the difficulty to factorize large numbers

RSA Algorithms

Key Generation

Select p, q	p and q both prime, $p \neq q$
Calculate $n = p \times q$	
Calculate $\phi(n) = (p - 1)(q - 1)$	
Select integer e	$\gcd(\phi(n), e) = 1;\ 1 < e < \phi(n)$
Calculate d	$d \equiv e^{-1} \pmod{\phi(n)}$
Public key	$PU = \{e, n\}$
Private key	$PR = \{d, n\}$

Encryption

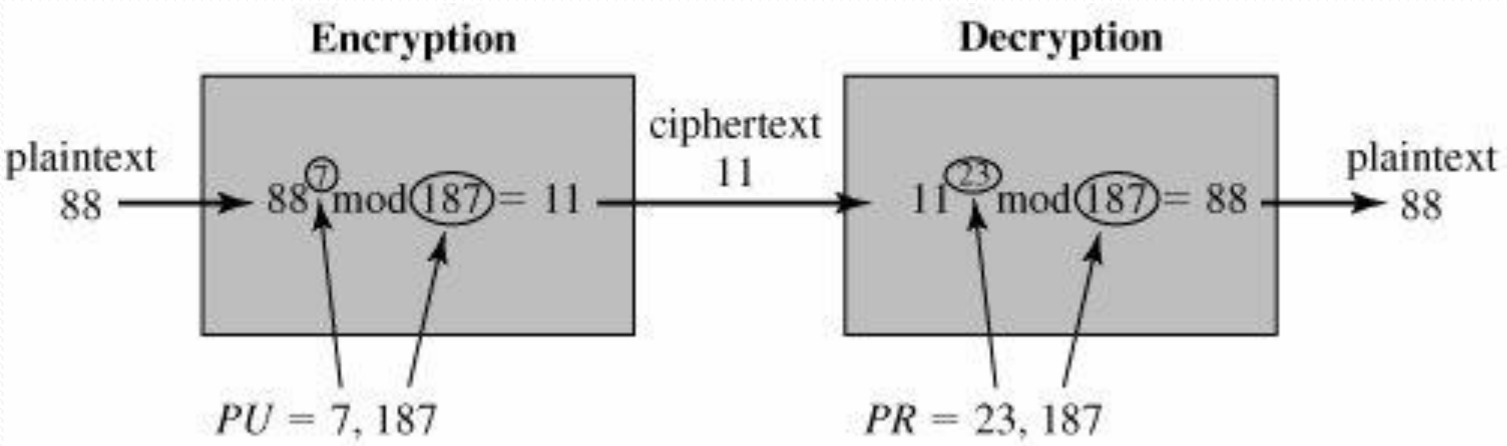
Plaintext:	$M < n$
Ciphertext:	$C = M^e \bmod n$

Decryption

Ciphertext:	C
Plaintext:	$M = C^d \bmod n$

Example

- p=17, q=11
- n=pq=187
- $\Phi(n)=(p-1)*(q-1)=160$
- e is prime less than $\Phi(n)$ and $GCD(\Phi(n),e)=1$ (e.g. 7)
- $d=e^{-1} mod\ \Phi(n)=23$ ($23*7=161$)



How to Break RSA?

1. Factorize n = Find p and q.
2. Find $\Phi(n)=(p-1)*(q-1)$
3. Find $d=e^{-1} \bmod \Phi(n)$

Now you have the private key!!!!

The only problem is that it is mathematically very difficult to factorize n.

Diffie-Hellman

- Published by Diffie and Hellman in 1976
- First Public Key algorithm
- Can be used only for key exchange
- Depends on the difficulty to calculate discrete logarithms

What is a discrete logarithm?

- *a* is a primitive root of a prime number *p* iff its powers generate all numbers from *1* to *p-1*.
- $a \bmod p, a^2 \bmod p, a^3 \bmod p, \ldots\ldots, a^{p-1} \bmod p$ equal *1, 2, ...,* *p-1* in some permutation.
- For every integer $b < p$ and a primitive root *a* of the prim *p* there exist a unique number *i* where:

  $b = a^i \bmod p$ where $0 \le i \le (p-1)$
- Discrete logarithm $dlog_{a,p}(b)=i$ where $b = a^i \bmod p$
- This is difficult and slow!!

Diffie-Hellman

- The point is that users A and B will be able to calculate the secret key using only:
  1. His private key
  2. Other's public key

- Eve needs to do a discrete logarithm because she does not have any of the private keys.

Global Public Elements

q	prime number
α	$\alpha < q$ *and* α a primitive root of q

User A Key Generation

Select private X_A	$X_A < q$
Calculate public Y_A	$Y_A = \alpha^{X_A} \bmod q$

User B Key Generation

Select private X_B	$X_B < q$
Calculate public Y_B	$Y_B = \alpha^{X_B} \bmod q$

Calculation of Secret Key by User A

$K = (Y_B)^{X_A} \bmod q$

Calculation of Secret Key by User B

$K = (Y_A)^{X_B} \bmod q$

Why it works?

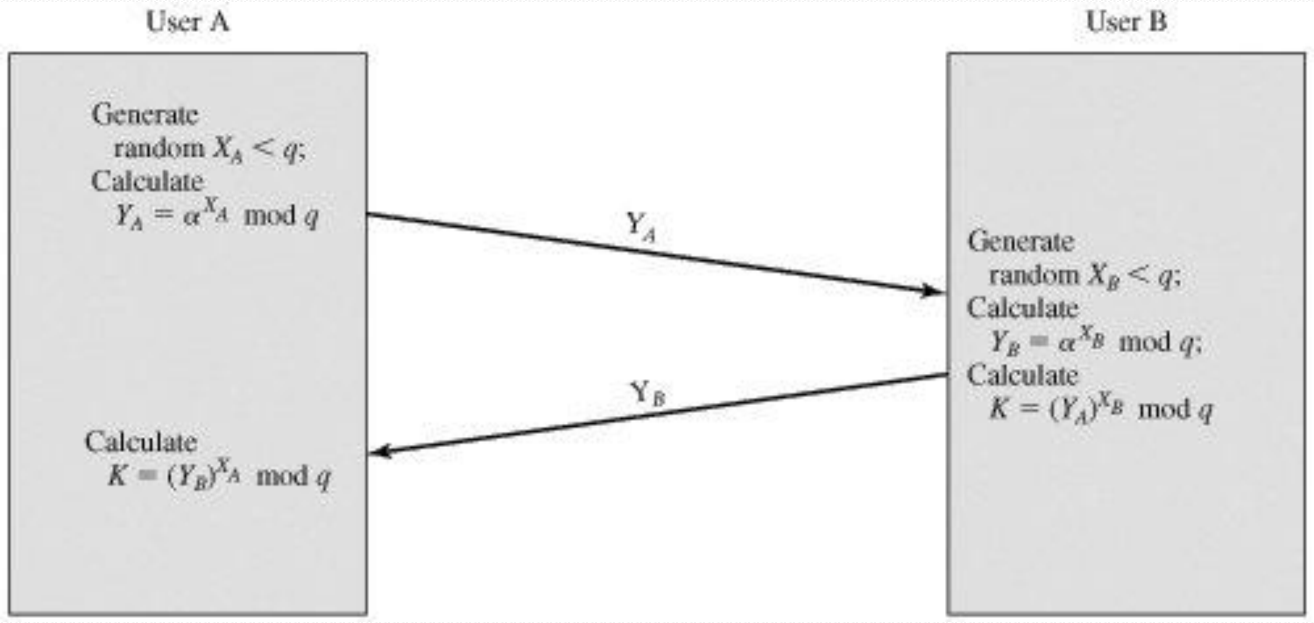
$$
\begin{aligned}
K &= (Y_B)^{X_A} \bmod q \\
&= (\alpha^{X_B} \bmod q)^{X_A} \bmod q \\
&= (\alpha^{X_B})^{X_A} \bmod q && \text{by the rules of modular arithmetic} \\
&= (\alpha^{X_B X_A} \bmod q \\
&= (\alpha^{X_A})^{X_B} \bmod q \\
&= (\alpha^{X_A} \bmod q)^{X_B} \bmod q \\
&= (Y_A)^{X_B} \bmod q
\end{aligned}
$$

Numeric example

- $q=71$
- $\alpha=7$
- $X_A=5$
- $X_B=12$
- $Y_A=7^5 mod71=51$
- $Y_B=7^{12} mod71=4$

- $K=4^5 mod71=51^{12} mod71=30$

Key exchange using Diffie-Hellman



- Can be broken using Man-in-the-Middle Attack

Man-in-the-Middle Attack

$$A \rightarrow E : Y_A$$

$$E \rightarrow B : Y_{D1}$$

$$\begin{cases} E : K_2 = Y_A{}^{X_{D2}} \bmod q \\ B : K_1 = Y_{D1}{}^{X_B} \bmod q \end{cases}$$

$$B \rightarrow E : X_A$$

$$E \rightarrow A : Y_{D2}$$

$$\begin{cases} E : K_1 = Y_B{}^{X_{D1}} \bmod q \\ A : K_2 = Y_{D2}{}^{X_A} \bmod q \end{cases}$$

Now E has K_1 shared with B and
K_2 shared with A

A and B think that they share the key with each other

$$A \rightarrow E : E(K_2; M)$$

$$E \rightarrow B : E(K_1; M)$$

or

$$E \rightarrow B : E(K_1; M')$$

Other Public Key systems

- Digital Signature Standard (DSS)
  - Only for signature

- Elliptic Curve Cryptography (ECC)
  - General Purpose Public Key Encryption Algorithm
  - More difficult to understand than RSA
  - Provides similar security for smaller key size
  - Not tested as much as RSA

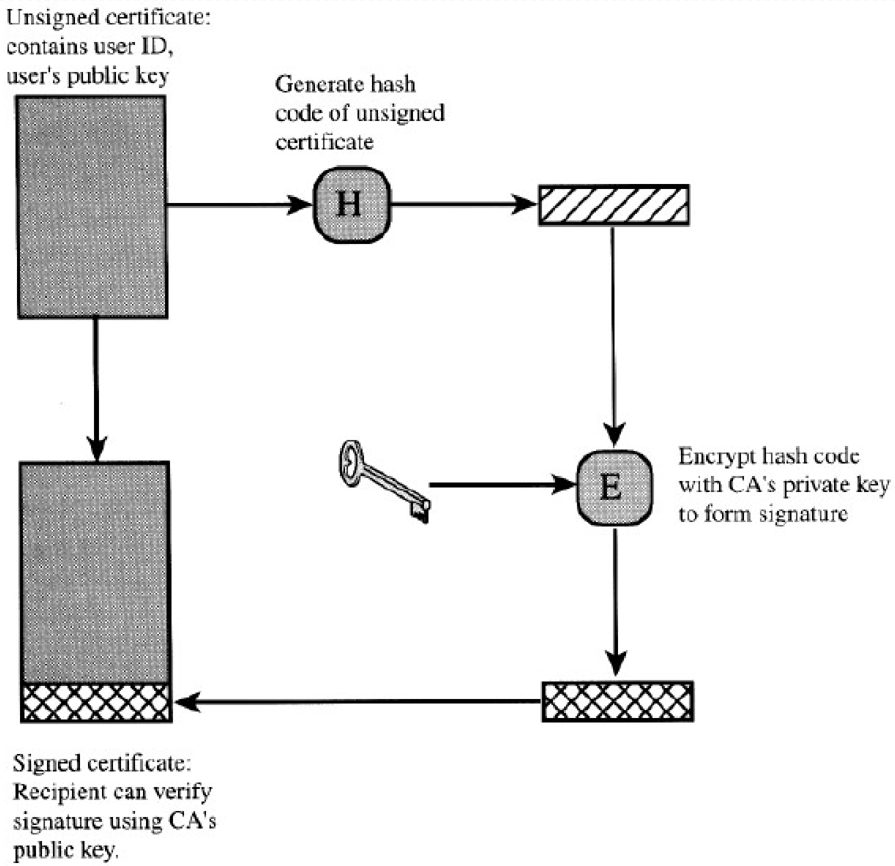
Digital Signatures

1. Encrypt Whole message
   - $C=Ep(Pr_A:M)$
   - C and M must be kept to prove the signature
   - C provides NO confidentiality. Why?

2. Encrypt an authenticator
   - $C=Ep(Pr_A:H(M))$
   - Only M and H(M) need to be kept
   - H used is usually SHA-1
   - No confidentiality. Why?

Distribution of Public Keys

- Public Key Certificates
- CA=Certification Authority
- CA's sign public keys of users with its private key
- X.509 standard
- Used in SSL, Secure Electronic Transaction (SET), S/MIME



Distribution Shared Keys

1. Use Diffie-Hellman

2. Use Public Key Encryption (Like RSA or ECC)

$$A \rightarrow B : E(k, M) + E_p\left(K_B^{pub}, k\right)$$

Can you see any problem in this exchange in terms of authentication?