Statistics and Noise 1

1. Prove that the standard deviation of a square signal with zero mean equals half its peak to peak value.

2. Find the relation between the acquired signal variance and RMS value for zero mean signals.

3. Using the definition: $\sigma^2 = \frac{1}{N-1}\sum_{i=0}^{N-1}(x_i-\mu)^2$ prove that: $\sigma^2 = \frac{1}{N-1}\left[\sum_{i=0}^{N-1} x_i^2 - \frac{1}{N}\left(\sum_{i=0}^{N-1} x_i\right)^2\right]$. What is the advantage of the second method for calculating the variance?

4. You have a sensor that reads the temperature of a patient 20 times every second. You recorded 20 seconds in a log file. Write a simple program to calculate the SNR and CV of this signal. Assume that there is a function *GET_VALUES* that can read the log file.

5. Show that the acquired signal variance and underlying process variance become the same when the number of samples becomes infinite.

6. Calculate the mean and variance for the first 11 samples of the digitized version of the following signals (assume the sampling period is 0.05 seconds):
   a. $x(t) = e^{-3t}\cos(2\pi t)$
   b. $x(t) = 2\cos(10t) - 3\sin(2t)$

7. Given that the mean of the signal x[n] is μ and its standard deviation is σ, find the mean and standard deviation of the following signals:
   a. y[n]=5x[n]
   b. y[n]=3x[n-1]
   c. y[n]=2x[n]+4x[n-1]
   d. y[n]=3x[n]-5

8. You have a sensor that always reads less than the actual value of its measured quantity. Is this a precision or accuracy problem?

9. To decrease the expected difference between the measured and true value of some quantity to half its value, how much do you need to increase the number of measurements?