Linear Systems

1. True or False and why?
   a. All linear systems are statically linear
   b. All statically linear systems are linear
   c. To test linearity we need only check homogeneity
   d. All linear systems are shift invariant
   e. If the system A has sinusoidal fidelity property then it is linear
   f. All linear systems are commutative
2. Check the following systems for linearity, homogeneity, shift invariance, sinusoidal fidelity and addictiveness (if there are constants find the range within which the system is linear if any):
   a. $y[n]=g[n]x[n]$ where g[n] is given
   b. $y[n]=\sum_{k=n_0}^{n} x[k]$
   c. $y[n]=\sum_{y=n-n_0}^{n+n_0} x[k]$, n_0 is a known constant
   d. $y[n]=e^{x[n]}$
   e. $y[n]=x[-n+n_0]$, n_0 is a known constant
   f. $y[n]=y[n-2]-0.5y[n-1]+1.2x[n]$
   g. $y[n]=x[n-1]\sin\frac{\pi n}{4}$
3. Show that for any linear system, if the input is all zeros, the output is all zeros.
4. Show that if a system is homogeneous then: if $x[n]\rightarrow y[n]$ then $ax[n]\rightarrow ay[n]$ when a is an integer larger than zero. Does this prove addictiveness?
5. Using the fact that all homogeneous additive systems are linear, show that the systems described by the following equations are linear then check if they are shift invariant and linearize the system if it was not linear:
   a. $y[n]=\sum_{k=n_0}^{n_1} a[k]x[n+k]$, where $n_0, n_1, a[k]$ are constants, $n_0<0, n_1\geq 0$
   b. $y[n]=\frac{x[n-1]}{2x[n+n_0]}$, where n_0 is a constant